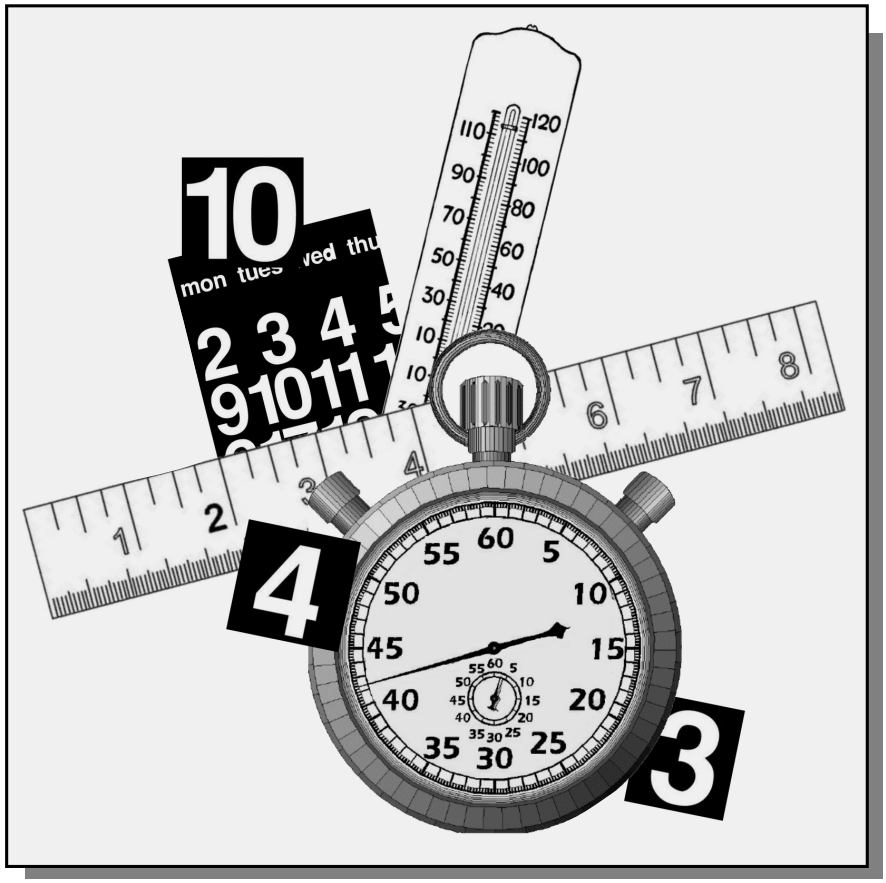
Intermediate Math Skills



Chapter 2

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Intermediate Math Skills

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"The study of Mathematics cannot be replaced by any other activity that will train and develop man's purely logical faculties to the same level of rationality."

—C. O. Oakley



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Intermediate Math Skills

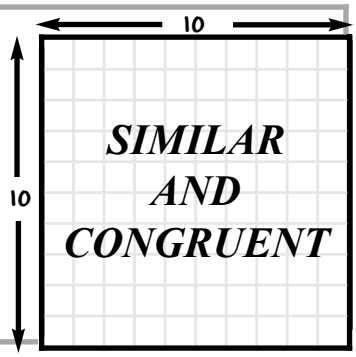
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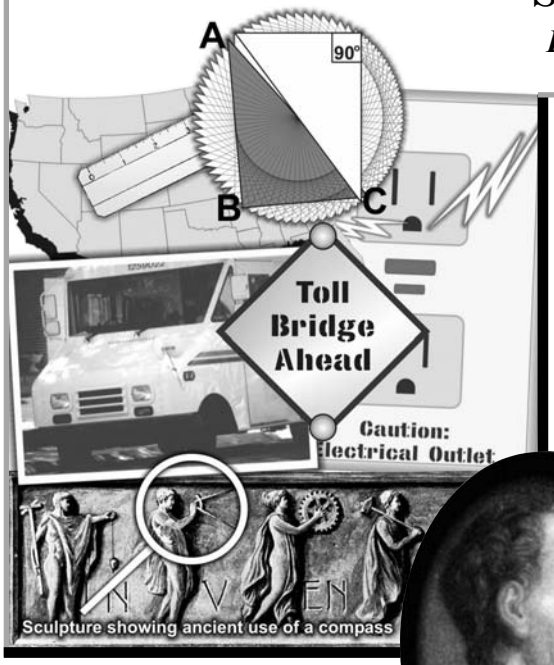
Lesson 1

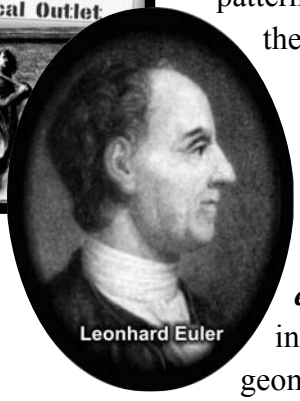
Chapter 2

Section 1

Lessons 1-5







Professor K reminds you that objects which are ***similar*** have the same shape, but not necessarily the same size. ***Congruent*** objects have the same size and shape. In many paintings, cloth prints and ***tapestries***, similar and congruent shapes are used in patterns to make something that is pleasing to the eye of the beholder.



Professor K

The Greeks used similar and congruent shapes extensively in their mathematical calculations and proofs. The basic shapes of the Greeks were the *circle* and the *straight line*. The Greeks used a ***straight edge*** and a ***compass*** as their basic instruments for the construction of geometric shapes. The Greeks used the *straight edge* to draw lines and the *compass* to draw circles, construct angles and measure distances to make congruent objects.

In this lesson, Professor K will show you how similar shapes can also be congruent and introduce some new properties with respect to triangles.

Look at **Example Sets 1 and 2** as Professor K explains the Side-Angle-Side property listed below.

VOCABULARY

Similar (~): objects that have the same shape
Congruent (≅): having the same shape and size
Tapestries: thick fabric with a picture woven in, used as wall hangings or furniture coverings
Straight edge: an instrument similar to a ruler used to draw straight lines
Compass: an instrument used to draw circles and curved lines called arcs
Hash marks: lines on triangles to indicate corresponding sides or angles to establish congruency

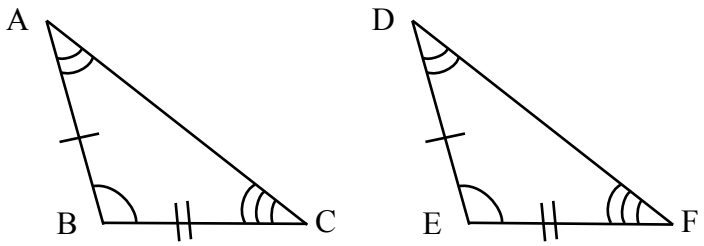
Side-Angle-Side Property of Triangles (S-A-S)

If two corresponding sides and the angle that is included between them of one triangle are congruent to two sides and the angle that is included between them of another triangle, then the two triangles are congruent.

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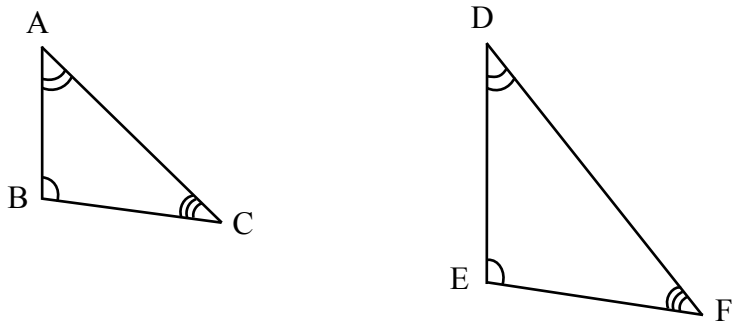
Example Set 1

In the triangles below AB ≅ DE, BC ≅ EF, ∠ABC ≅ ∠DEF, ∠BAC ≅ ∠EDF and ∠ACB ≅ DFE.



△ABC ~ △DEF because all three corresponding angles are congruent (A-A-A). △ABC ≅ △DEF. This statement of congruency is also true for the two triangles because of the Side, AB ≅ DE; the included Angle, ∠ABC ≅ ∠DEF; and Side, DE ≅ EF. Since two corresponding sides and the included angle are congruent, then the Side-Angle-Side property forces the similar triangles to be congruent as well. Professor K wants you to notice that the ***hash marks*** show you the corresponding sides and angles.

Example Set 2



In the triangles above, ∠ABC ≅ ∠DEF, ∠BAC ≅ ∠EDF and ∠ACB ≅ ∠DFE. Therefore, △ABC ~ △DEF because all three corresponding angles are congruent (A-A-A property).

△ABC and △DEF are not congruent even though all three angles are congruent, since none of the corresponding sides are congruent.

Another property used to determine if two triangles are congruent is the Side-Side-Side property. This property is stated below and Professor K shows you how to use the property in **Examples Sets 3 and 4**.



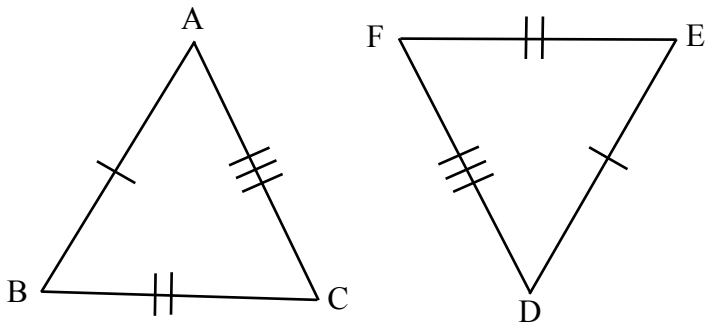
SIDE-SIDE-SIDE PROPERTY (S-S-S)

If three corresponding sides of one triangle are congruent to three corresponding sides of another triangle, then the two triangles are congruent.



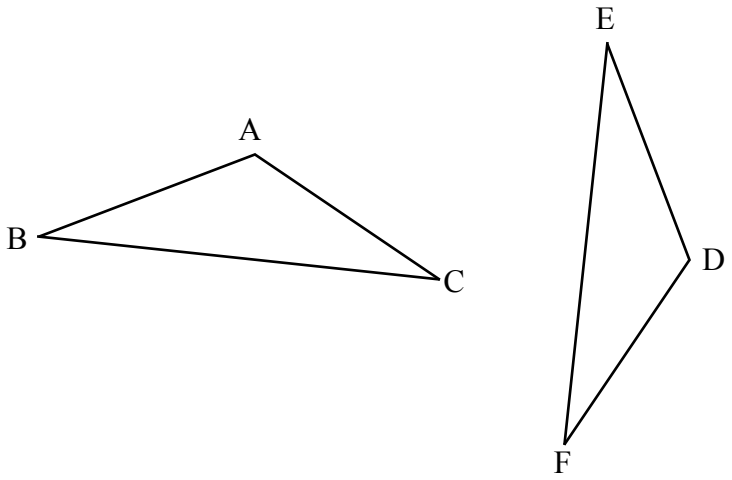
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Example Set 3



In the triangles above, AB ≅ DE, AC ≅ DF and BC ≅ EF. Since all three corresponding sides are congruent in △ABC and △DEF, then △ABC ≅ △DEF.

Example Set 4



In the triangles above, AB ≅ DE, AC ≅ DF and BC ≅ EF. Since all three corresponding sides are congruent in △ABC and △DEF, then △ABC ≅ △DEF.

The next property of congruence for triangles is the Angle-Side-Angle property. Study this property stated below along with **Examples Sets 5 and 6** to understand it fully.



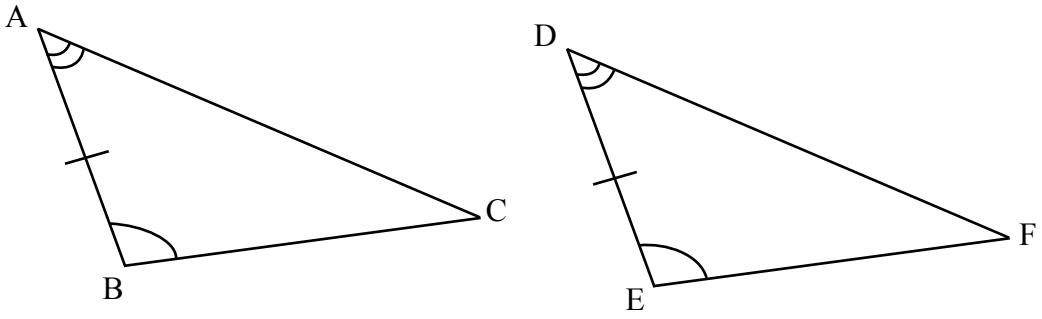
ANGLE-SIDE-ANGLE PROPERTY (A-S-A)

If two corresponding angles and the included side of one triangle are congruent to two corresponding angles and the included side of another triangle, the two triangles are congruent.

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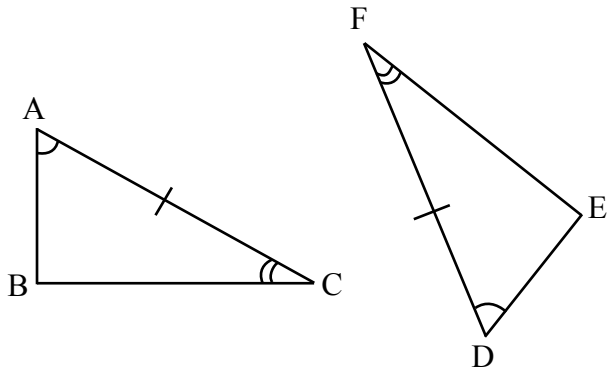
Example Set 5

Look closely at the triangles below to see how two triangles are congruent using the A-S-A property.



In $\triangle ABC$ and $\triangle DEF$, $\angle BAC \cong \angle EDF$, $AB \cong DE$ and $\angle ABC \cong \angle DEF$. The triangles are congruent using the A-S-A property because two corresponding angles and the line segment (side) between them are all correspondingly congruent.

Example Set 6



In the triangles above $\angle ACB \cong \angle DFE$, $AC \cong DF$ and $\angle CAB \cong \angle FDE$. Since these two triangles contain two congruent corresponding angles and a congruent side between them, then the triangles are congruent.

Show Professor K that you understand the properties of congruent triangles by working the activities for this lesson.

Life Principle



"Hide not your talents."

—Benjamin Franklin

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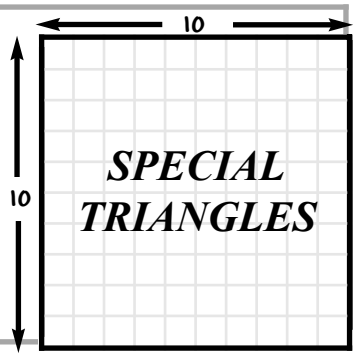
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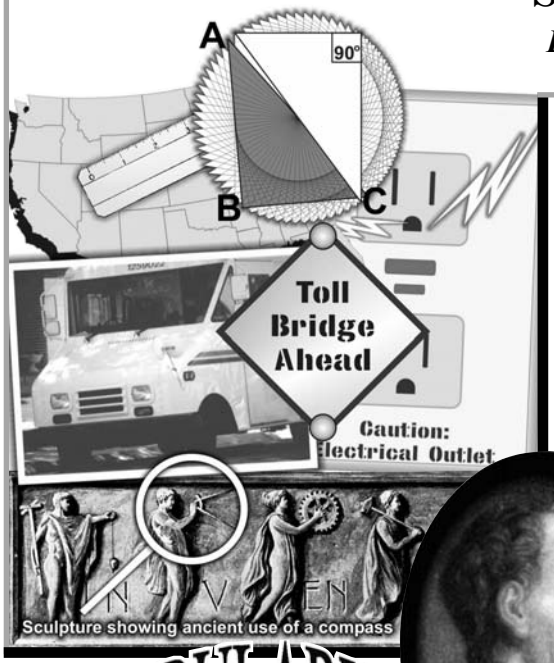
Geometry

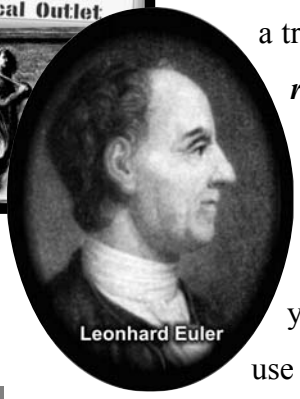
Chapter 2

Section 1

Lessons 1-5







You have probably heard someone was given special treatment or not punished for wrongdoing because theirs was a "special case." Professor K has heard these circumstances too, but he believes right is right and wrong is wrong. Some things are always right. For example, a triangle containing an angle of 90° is a ***right triangle***, and a triangle that contains three angles of less than 90° is an *acute triangle*.

In this lesson Professor K will show you two *special case* triangles and how to use the characteristics of those triangles to determine the dimensions of the sides when only one dimension is known.

Professor K offers the following property concerning ***30 - 60 - 90 triangles***. Study this property and look at **Example Sets 1, 2** and **3** as Professor K explains the property more fully and how the Pythagorean Theorem is used to work with these triangles.

VOCABULARY

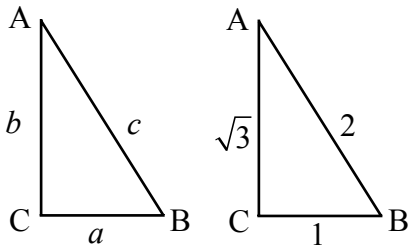
Right triangle: a triangle containing a right angle
30 - 60 - 90 triangle: a triangle containing angles of measures 30°, 60° and 90°
Hypotenuse: the side of a right triangle opposite the 90° angle; the longest side of a right triangle
Leg: side of a right triangle that is not the hypotenuse
45 - 45 - 90 triangle: a triangle containing angles of measures 45°, 45° and 90°
Isosceles right triangle: a right triangle containing two congruent sides and two congruent angles

30 - 60 - 90 Triangle Property

If the other two angles of a right triangle are 30° and 60°, then the ***hypotenuse*** is twice the length of the shortest ***leg*** of the triangle (the side opposite the 30° angle). The other leg (the side opposite the 60° angle) is $\sqrt{3}$ times longer than the shortest leg.

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Example Set 1



The triangle (△ABC) above is a 30 - 60 - 90 triangle with m∠ACB = 90°, m∠ABC = 60° and m∠BAC = 30°. (Remember, m∠ means the measure of the angle.) The second triangle above identifies the dimensions of the legs of △ABC, $a = 1$, $b = \sqrt{3}$ and $c = 2$.

Professor K substitutes each of the values $a = 1$, $b = \sqrt{3}$ and $c = 2$ into the Pythagorean Theorem, $a^2 + b^2 = c^2$.

$$1^2 + (\sqrt{3})^2 = 2^2$$

Professor K then performs the calculations shown in the resulting equation.

$$1^2 = (1 \bullet 1) = 1$$
$$(\sqrt{3})^2 = (\sqrt{3} \bullet \sqrt{3}) = 3$$
$$2^2 = 2 \bullet 2 = 4$$

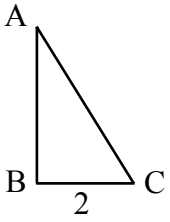
Professor K substitutes these calculations back into the equation to obtain:

$$1^2 + (\sqrt{3})^2 = 2^2 \rightarrow 1 + 3 = 4.$$

Professor K performs the calculation 1 + 3 to obtain 4 on the lhs (left hand side) of the equation. Professor K then sees that 4 = 4 is the resulting equation. Since this is a true statement, Professor K has shown that the 30 - 60 - 90 Triangle Property is true.

Example Set 2

The triangle below is a 30 - 60 - 90 triangle. The length of leg BC is shown as 2 units. Professor K will demonstrate how to find the length of the other two legs using the 30 - 60 - 90 Triangle Property.



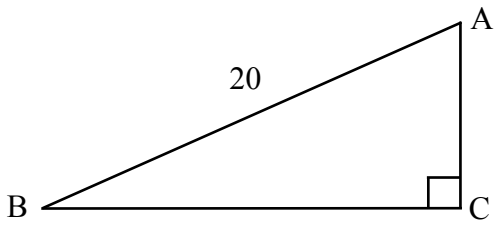


In △ABC, the length of the leg BC is two units. Using the 30 - 60 - 90 Triangle Property, Professor K calculates the length of the hypotenuse, AC to be twice the shortest leg, or 2 • 2 = 4 units. The length of the leg AB is calculated by multiplying the length of the shortest leg, BC by $\sqrt{3}$. The length of AB would be $2 \bullet \sqrt{3} = 2\sqrt{3}$ units.

Professor K wants you to notice that you do not need to calculate the value of $\sqrt{3}$, but you can leave it in radical (square root) form. However, if needed you can estimate the value of $\sqrt{3}$ to be approximately 1.73. Therefore, the length of AB would be approximately 3.46 units.

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Example Set 3



In △ABC, side AB is the hypotenuse. The length of side AB is 20 units. Professor K calculates the lengths of the legs of the triangle as shown below.

Using the 30 - 60 - 90 Triangle Property, Professor K calculates the length of leg, AC first. The shortest leg (the leg opposite the 30° angle) is one-half the hypotenuse, because the hypotenuse is twice the shortest leg. Therefore, AC = 20 ÷ 2 = 10 units.

Using the 30 - 60 - 90 Triangle Property, Professor K calculates the length of leg BC by multiplying the length of the shortest leg, AC by $\sqrt{3}$. Therefore, the length of BC = $10 \bullet \sqrt{3} = 10\sqrt{3}$ units.

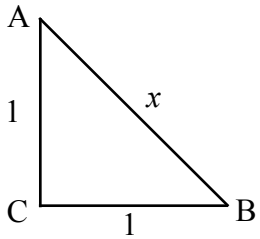
Another special case triangle is the ***45 - 45 - 90 triangle property*** shown below. Professor K shows you how the property is determined and how to use the property in **Example Sets 4, 5** and **6**.

45 - 45 - 90 Triangle Property

The hypotenuse of an ***isosceles right triangle*** has a length equal to $\sqrt{2}$ times the length of the one of the legs.

Professor K will use the Pythagorean Theorem in **Example Set 4** to show you that $\sqrt{2}$ is the multiplier needed to determine the length of the hypotenuse of a 45 - 45 - 90 triangle. Professor K introduced the Pythagorean Theorem to you in Chapter 4, Lesson 5. If you need to refresh your memory, be sure to look back at the lesson on how to use the Pythagorean Theorem.

Example Set 4



△ABC is an isosceles right triangle with m∠ACB = 90°, m∠CBA = m∠BAC = 45°. The length of legs BC and AC each equals 1 unit. The length of the hypotenuse AB is represented by the letter, *x*. To determine the value of *x* (the length of the hypotenuse) Professor K uses the Pythagorean Theorem, $a^2 + b^2 = c^2$. Professor K substitutes the values of $a = 1$ and $b = 1$ into the Pythagorean Theorem.

$$1^2 + 1^2 = c^2$$

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Professor K then performs the calculation shown in the resulting equation.

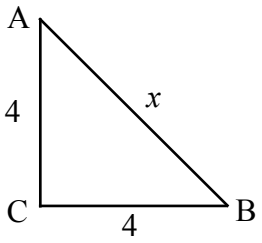
$$1^2 = (1 \bullet 1) = 1$$
$$1^2 = (1 \bullet 1) = 1$$

Professor K substitutes these calculations back into the equation to obtain:

$$1^2 + 1^2 = c^2 \rightarrow 1 + 1 = c^2.$$

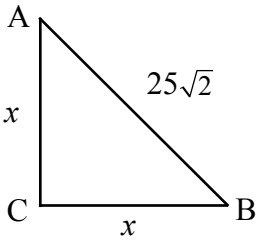
Professor K performs the calculation 1 + 1 to obtain 2 on the lhs of the equation. Therefore, $2 = c^2$. Professor K performs the operation of square root on both sides of the resulting equation. $\sqrt{2} = \sqrt{c^2}$. $\sqrt{c^2} = c$, therefore, $c = \sqrt{2}$. Professor K has determined that the hypotenuse of $\triangle ABC$ is $\sqrt{2}$ units.

Example Set 5



In the isosceles right triangle above, we will determine the value of x.
Professor K uses the 45 - 45 - 90 Triangle Property to determine the length of the hypotenuse.
Professor K multiplies the length of one of the legs, 4 units times $\sqrt{2} \rightarrow 4 \bullet \sqrt{2} = 4\sqrt{2}$.
Therefore, the value of x (length of the hypotenuse) is $4\sqrt{2}$ units.

Example Set 6



In the isosceles right triangle above, we will determine the value of x.
Professor K uses the 45 - 45 - 90 Triangle Property to determine the length of the leg, CB.
Professor K divides the length of the hypotenuse by $\sqrt{2} \rightarrow 25\sqrt{2} \div \sqrt{2} = {}^{(25\sqrt{2})}/_{\sqrt{2}} = 25$.
Therefore, the value of x (length of the leg, CB) is 25 units.

Show Professor K that you understand the properties of special case triangles by working the activities for this lesson.

Life Principle: "You are always right to refuse to do wrong."
—Ronald E. Johnson, Ph. D.



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Intermediate Math Skills

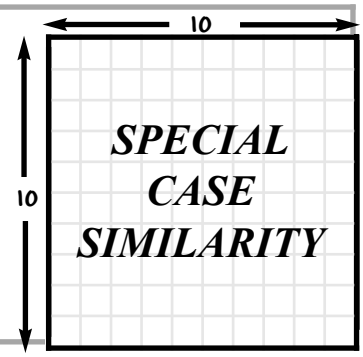
Lesson 3

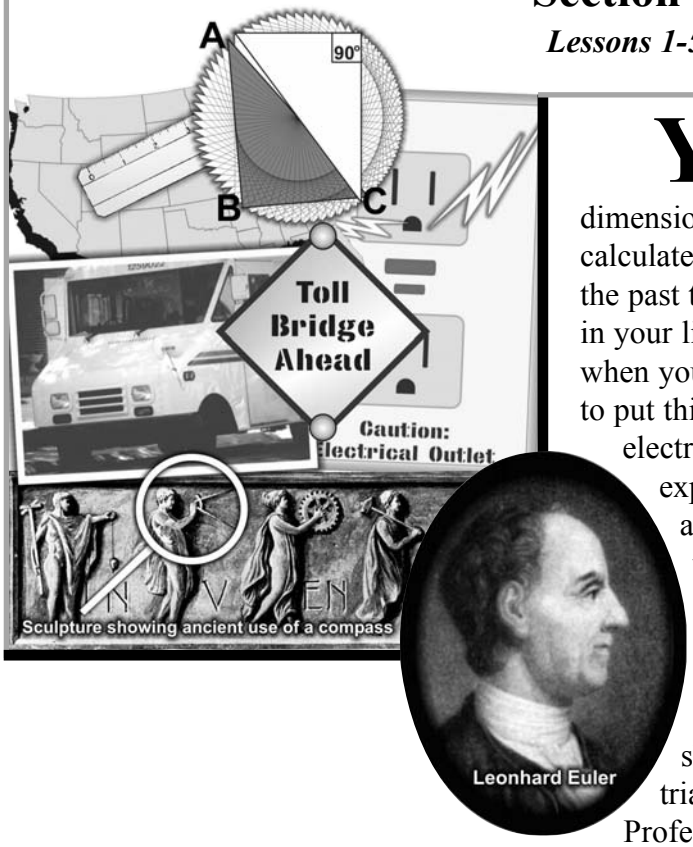
Geometry

Chapter 2

Section 1

Lessons 1-5





You probably remember in an earlier lesson (Chapter 7, Lesson 3) that the dimensions of similar triangles were compared and calculated. You also can use similar situations from the past to stay away from harm or solve problems in your life today. Professor K reminds you that when you were younger your parents told you not to put things, especially metal objects, into electrical outlets. If you disobeyed, the harsh experience was such that you did not do it again. In a similar situation, when adults who have authority over you say not to do a thing because it may harm you, you should compare previous experiences and obey the rules for your own well-being.

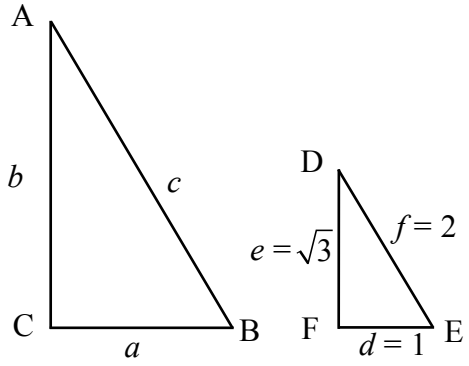
In this lesson, Professor K will show you how to compare *special case* triangles and calculate unknown dimensions. Professor K reminds you that ***ratios*** and ***proportions*** can be set up between the sides of similar triangles. Look at **Example Sets 1 and 2** as Professor K explains how to use the similarity between special case triangles to determine the dimensions of another triangle.

VOCABULARY

Ratios: comparison of two numbers, usually written in fraction form

Proportion: a comparison of two ratios written as an equality

Example Set 1



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△ABC (previous page) is a 30 - 60 - 90 triangle with m∠ACB = 90°, m∠ABC = 60° and m∠BAC = 30°. △DEF is also a 30 - 60 - 90 triangle with m∠DFE = 90°, m∠DEF = 60° and m∠EDF = 30°. Calculate the value of *a, b,* and *c* using the properties of similar triangles. Assume that the value of *c* is twice that of the corresponding side of △DEF.

Step 1

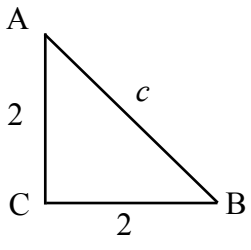
Professor K determines the value of *c* to be 4 units because it is twice the length of DE. ($2 \bullet 2 = 4$)

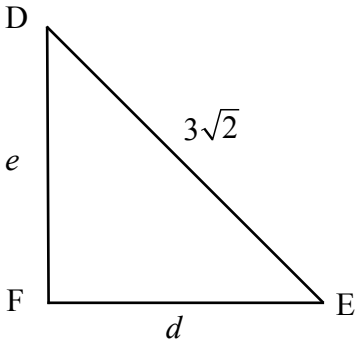
Step 2

The instructions stated to use the properties of similar triangles to calculate the dimensions of △ABC. Therefore, Professor K sets up the following ratios and proportions to compare the lengths of the corresponding sides of triangles △ABC and △DEF. $^{f}/_{c} = {}^{d}/_{a} \rightarrow {}^{2}/_{4} = {}^{1}/_{a}$. Professor K replaces the values for the lengths of four sides of the triangles into a ratio and proportion statement as shown. Professor K cross-multiplies to obtain $2 \bullet a = 4 \bullet 1 \rightarrow 2a = 4$. Professor K divides by two, ${}^{2a}/_{2} = {}^{4}/_{2}$, to determine that $a = 2$. Then, $^{f}/_{c} = {}^{e}/_{b} \rightarrow {}^{2}/_{4} = {}^{\sqrt{3}}/_{b}$. Professor K replaces the values for the lengths of four sides of the triangles into a ratio and proportion statement as shown. Professor K cross-multiplies to obtain $2 \bullet b = 4 \bullet \sqrt{3} \rightarrow 2b = 4\sqrt{3}$. Professor K divides by two, ${}^{2b}/_{2} = {}^{4\sqrt{3}}/_{2}$, to determine that $b = 2\sqrt{3}$.

In **Example Set 1**, Professor K calculated the dimensions of △ABC to be $a = 2$ units, $b = 2\sqrt{3}$ units and $c = 4$ units. Professor K checks his calculations using the 30 - 60 - 90 Triangle Property. One-half of the hypotenuse, *c* of △ABC, is $4 \div 2 = 2$. Therefore, according to the 30 - 60 - 90 Triangle Property the value of *a* is *2* and the length of CB is 2 units. The value of *b* is $2 \bullet \sqrt{3}$ or $2\sqrt{3}$, so the length of AC is $2\sqrt{3}$ units. These calculations show that Professor K was correct in his calculations of the dimensions of △ABC using the properties of similar triangles.

Example Set 2







Triangles ABC and DEF above are isosceles right triangles. Calculate the missing dimensions using the properties of similar triangles. Check your answer using the 45 - 45 - 90 Triangle Property.

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Step 1

Professor K uses the 45 - 45 - 90 Triangle Property to determine the length of the hypotenuse, AB of $\triangle ABC$. Professor K multiplies the length of one of the legs, 2 units times $\sqrt{2}$, or $2 \bullet \sqrt{2} = 2\sqrt{2}$. Therefore, the value of c (length of the hypotenuse) is $2\sqrt{2}$ units.

Step 2

The instructions stated to use the properties of similar triangles to calculate the missing dimensions. Therefore, Professor K sets up the following ratios and proportions to compare the lengths of the corresponding sides of the triangles $\triangle ABC$ and $\triangle DEF$. $^{a}/_{c} = {}^{d}/_{f} \rightarrow {}^{2}/_{2\sqrt{2}} = {}^{d}/_{3\sqrt{2}}$. Professor K replaces the values for the lengths of four sides of the triangles into a ratio and proportion statement as shown. Professor K cross-multiplies to obtain $2\sqrt{2} \bullet d = 2 \bullet 3\sqrt{2} \rightarrow 2\sqrt{2}d = 6\sqrt{2}$. Professor K divides by $2\sqrt{2}$, $^{(2\sqrt{2}d)}/_{(2\sqrt{2})} = {}^{(6\sqrt{2})}/_{(2\sqrt{2})}$, to determine that $d = 3$. $^{b}/_{c} = {}^{e}/_{f} \rightarrow {}^{2}/_{2\sqrt{2}} = {}^{e}/_{3\sqrt{2}}$. Professor K replaces the values for the lengths of four sides of the triangles into a ratio and proportion statement as shown. Professor K cross-multiplies to obtain $2\sqrt{2} \bullet e = 2 \bullet 3\sqrt{2} \rightarrow 2\sqrt{2}e = 6\sqrt{2}$. Professor K divides by $2\sqrt{2}$, $^{(2\sqrt{2}e)}/_{(2\sqrt{2})} = {}^{(6\sqrt{2})}/_{(2\sqrt{2})}$, to determine that $e = 3$.

Professor K checks the calculations he performed in **Example Set 2** through similarity against the 45 - 45 - 90 Triangle Property. Professor K determines that his calculations are correct, because the length of DE, $3\sqrt{2}$, divided by $\sqrt{2}$ equals 3 units for the length of EF and DF in accordance with the 45 - 45 - 90 Triangle Property.

Show Professor K that you understand the properties of special case triangles by working the activities for this lesson.

LIFE PRINCIPLE

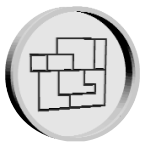


"Wise people learn from the past
to avoid harm in the present."
—Ronald E. Johnson, Ph. D.

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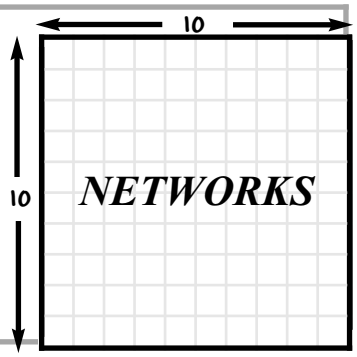
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Networks

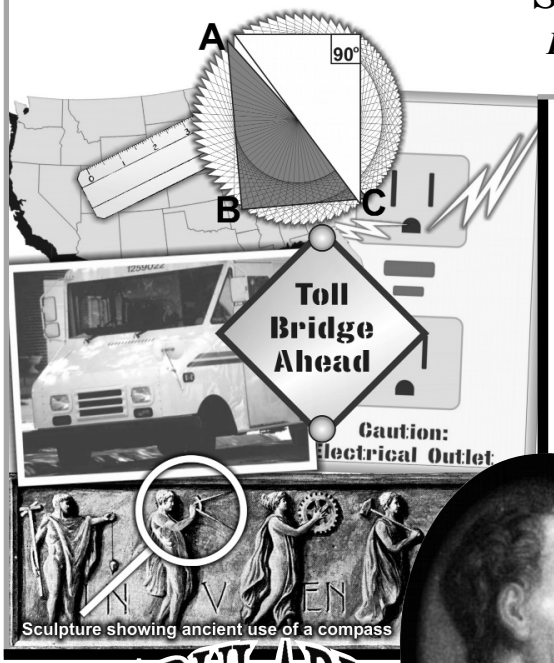
Lesson 4

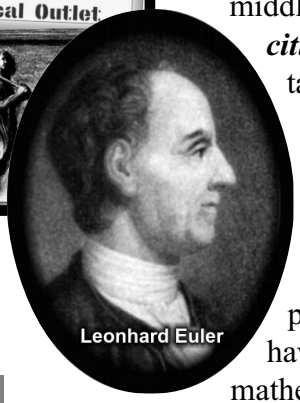
Chapter 2



Section 1

Lessons 1-5







Professor K

You have probably heard of ***networks*** in relation to computers that are interconnected. From ***Eighteenth Century*** world history, Professor K remembers reading about a city in Germany named, Kšnigsburg, through which a river flowed. At one particular location in the middle of the river were two islands. The ***citizens*** of the city of Kšnigsburg enjoyed taking walks along the river in the evenings. Seven bridges connected the two islands to the shore and to each other. As time passed, the people posed the problem among themselves as to whether a person could start walking from a particular point and return to that point, having crossed each bridge only once. A mathematician named Leonhard Euler solved the problem in the early 1700s by drawing the problem as a network. Remember that Professor K also believes drawing a picture of a problem can help you solve it. A diagram of the problem is shown in **Figure 4.1**.

VOCABULARY

Network: lines or paths that connect

Eighteenth Century: period of time consisting of the years 1700 A.D. to 1800 A.D.

Citizen: native or naturalized member of a city, town, state or country who owes allegiance to its government and is entitled to its protection

Vertex: a point at which lines intersect (plural is *vertices*)

Traversable: all paths can be traced without going over the same path twice

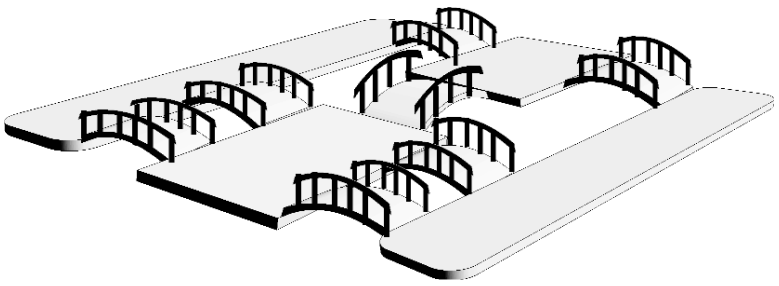


Figure 4.1

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Figure 4.2 is a drawing of the possible network that Euler might have drawn to illustrate the problem.

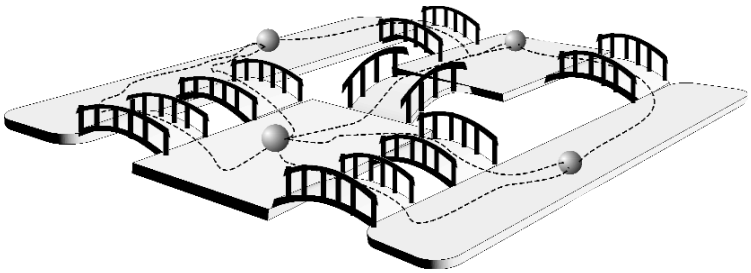


Figure 4.2

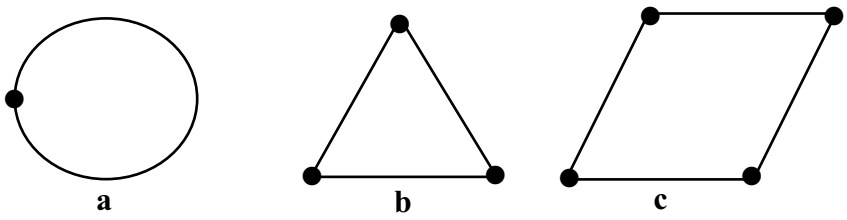
If you are able to follow a path completely around a network in a way that you start and end at the same point or another without traveling over (traversing) a path twice, the network is known as a Euler circuit. Each point (starting or stopping) is known as a ***vertex***. Professor K wants you to notice that the network in **Figure 8.2** has four vertices (represented by spheres) and seven paths. Professor K also points out that an odd number of paths meet at each vertex. Since the number of paths meeting at a vertex is odd, the vertex is known as an odd vertex. If the number of paths meeting at a vertex had been even, then the vertex would have been known as an even vertex. Mr. Euler determined that it was *not* possible to complete the Kšnigsburg problem network without traversing a path more than once. This problem followed the properties of networks shown below.

PROPERTIES OF NETWORKS

- If exactly two odd vertices are in a network, then you can complete the network without traversing a path twice. This network is ***traversable*** because no paths are traversed more than one time. Each odd vertex must be a starting or stopping point.
- If all the vertices of a network are even, the network is traversable, (you can complete the network without traversing a path twice) and the network is an Euler circuit. Any of the vertices can be a starting point and must also be the stopping point.

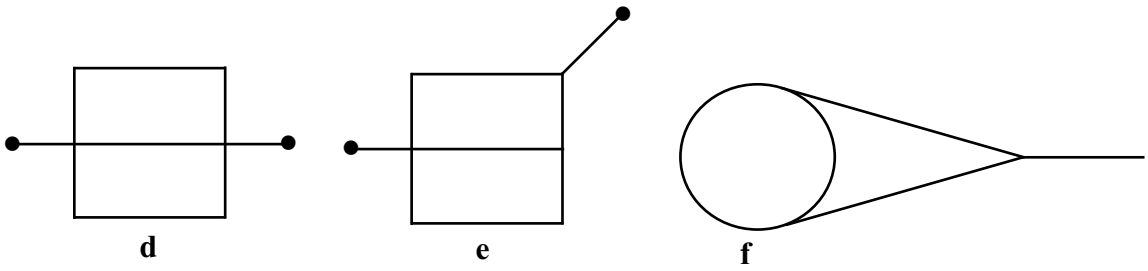
Look at **Example Set 1** to see how Professor K determines if it is possible to traverse the networks shown.

Example Set 1



Notice that all the vertices are even (two paths meet at each vertex) in networks **a**, **b** and **c**. Using the properties of networks, all three networks are traversable. All three are Euler circuits.

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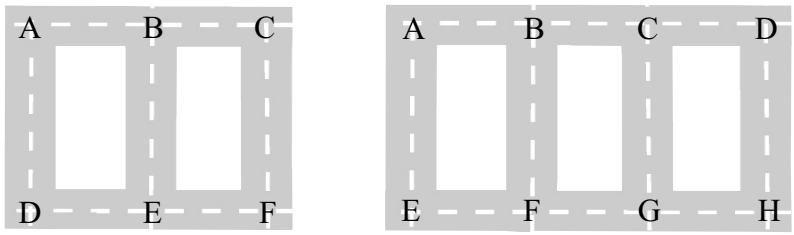


d **e** **f**

Notice in networks **d**, **e** and **f** that **d** has exactly two odd vertices, therefore **d** is traversable, but is not a Euler circuit because you cannot begin and end at the same point. Networks **e** and **f** have more than two odd vertices and are *not* traversable.

Delivery persons such as mailmen are interested in networks and especially Euler circuits. While making deliveries the mailman does not want to walk the same streets twice, and yet he wants to return to his vehicle. Look at the next example to see a map of city streets walked by a mailman. See how Professor K determines if the mailman's route is a Euler circuit and where he should park his vehicle.

Example Set 2



a **b**

Notice that network **a** is a traversable network because it contains exactly two odd vertices. The mailman could park his vehicle at either point B or E, however he would have to travel at least one street twice no matter were he parked his vehicle.

Network **b** is *not* traversable because this network contains more than two odd vertices (four).

Show Professor K that you understand the properties of networks by working the activities for this lesson.

Life Principle



"Working together helps everyone attain the goal."

—Anonymous

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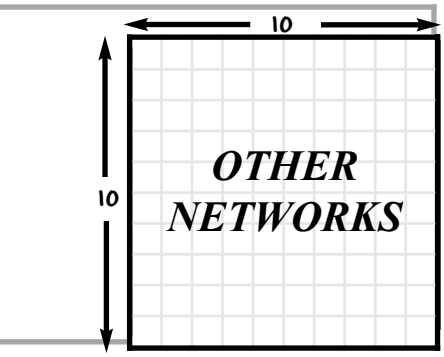
Networks

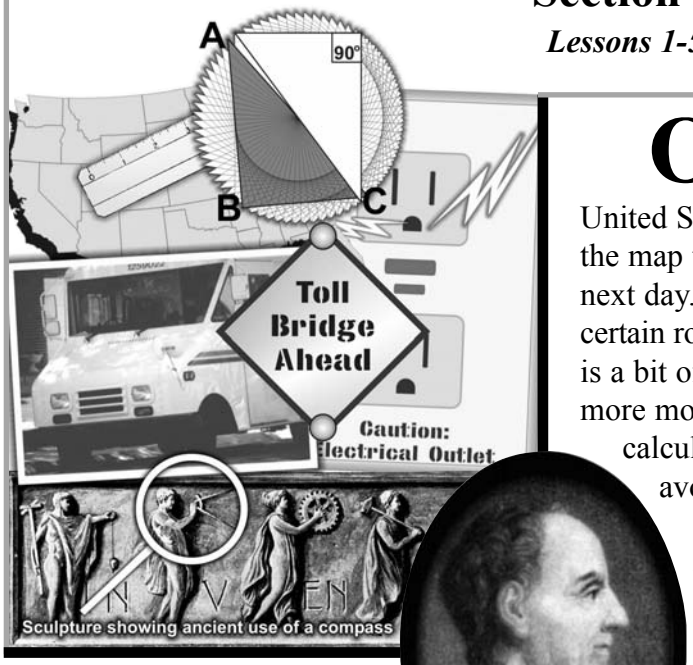
Lesson 5

Chapter 2

Section 1

Lessons 1-5





One summer, Professor K and his family took a vacation driving across the United States. Each night, Professor K looked at the map to plan the route that they would take the next day. Often, Professor K realized that along certain routes were ***toll*** roads or bridges. Professor K is a bit of a ***miser***, so he did not want to spend any more money than necessary. Therefore, he would calculate the time and gasoline necessary to avoid such toll roads, then compare costs.

The cost of moving from one point to another point is a problem that you may face each day. In this lesson, Professor K will present some simple problems that will illustrate how you determine the most efficient (cheapest or fastest) route to take, to reach a desired destination.

Look at the diagram in **Figure 5.1** of a portion of a map of city streets. Each intersection is labeled with a letter and a number representing the minutes consumed to proceed through that intersection.

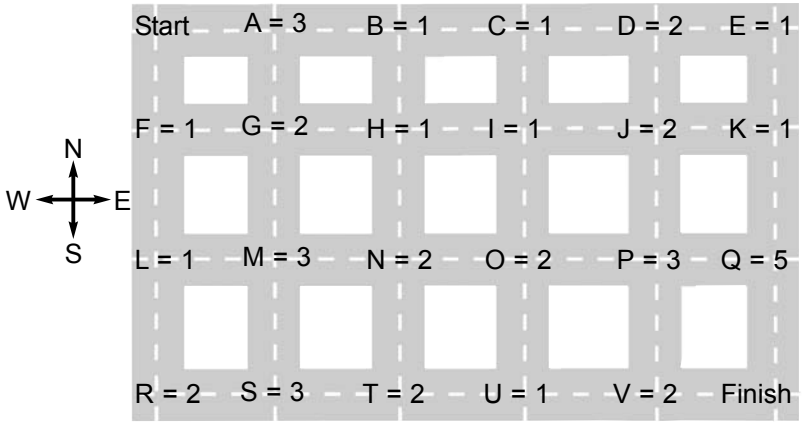


Figure 5.1

VOCABULARY

Toll: a fixed charge
Miser: a person who watches over his money very closely
Rule of thumb: a rule that is commonly accepted

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Professor K points out that just like in the real world, the time variations shown in **Figure 5.1** can be justified because of heavier or lighter traffic patterns. Look at **Example Set 1** as Professor K works through the problem presented.

RULE

When working network type problems involving value of travel along a path, a general ***rule of thumb*** from each new location is always to choose the next path with the lowest value.



Example Set 1

In **Figure 5.1** determine the fastest route from the *Start* to the *Finish*.

Step 1 Professor K considers this problem to be similar to a network, *a network of streets*. Professor K will start at a location and end at another. In this case Professor K does not have to cross over all paths as in a true network problem. Professor K can go in one of two directions when leaving "start." A general rule of thumb is always to take the smaller valued path. Therefore, from *Start* Professor K goes to point, *F*.

Step 2 Following the rule of thumb, Professor K proceeds to *L*, then to *R*. At *R*, Professor K has only one path to take and therefore, he goes to *S*, then to *T*, *U*, *V* and then to the *Finish*. As Professor K proceeds along the path, he writes a list of the values related to each intersection then finds the total "value," 1 + 1 + 2 + 3 + 2 + 1 + 2 = 12 minutes.

Step 3 Professor K checks to see if following the general rule of thumb is a restriction that provided the smallest total of minutes. Professor K looks at other routes and calculates the total travel minutes of other routes from *Start* to *Finish*.

1: Start A B C I O U V Finish = 3 + 1 + 1 + 1+ 2 + 1 + 2 = 11 minutes
2: Start F G H I O U V Finish = 1 + 2 + 1 + 1 + 2 + 1 + 2 = 10 minutes
3: Start F L M N T U V Finish = 1 + 1 + 3 + 2 + 2 + 1 + 2 = 12 minutes

Step 4 Professor K chooses route number 2 as the fastest route from *Start* to *Finish*.

Professor K points out that the general rule of thumb is the quickest method to obtain an answer; notice the numerous routes that can be taken. It would take more time to calculate all the possible routes than to proceed along a given route. However, Professor K quickly checked a few other routes and found that route 2 was quicker than the general rule of thumb route.

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Example Set 2

Look again at **Figure 5.1**. If Professor K wanted to take the most possible time getting from *Start* to *Finish*, what path would he take?

Professor K thinks about the problem at hand and decides to use the opposite of the rule of thumb to work out this problem. Instead of taking the path of lowest value from each location, he needed to take the path with the greatest value. Therefore, the path would look like this:
Start A G M S T N O P Q Finish = 3 + 2 + 3 + 3 + 2 + 2 + 2 + 3 + 5 = 25 minutes.

Often, restrictions are placed on the path that you can take. Look at **Example Set 3** as Professor K works the same problem as **Example Set 2**. NOTE: once he has traveled one direction, he cannot go back the opposite direction. Notice how the path is different than in **Example Set 2**.

Example Set 3

Look at **Figure 5.1**. Determine the path that will take the greatest amount of time. Once you have proceeded south you cannot go back north. Once you have proceeded east, you cannot go back west.
Beginning at *Start*, Professor K's path would look like the following:
Start A G M S T U V Finish = 3 + 2 + 3 + 3 + 2 + 1 + 2 = 16 minutes

---

Show Professor K that you understand how to solve value travel problems by working the activity for this lesson.

LIFE PRINCIPLE



"The opportunities in life are limitless."

—Anonymous

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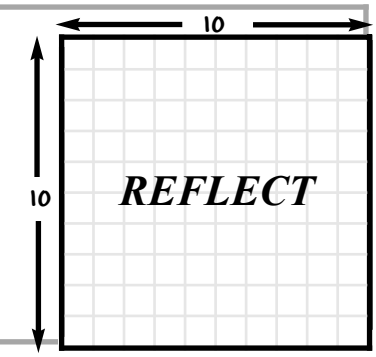
Geometry

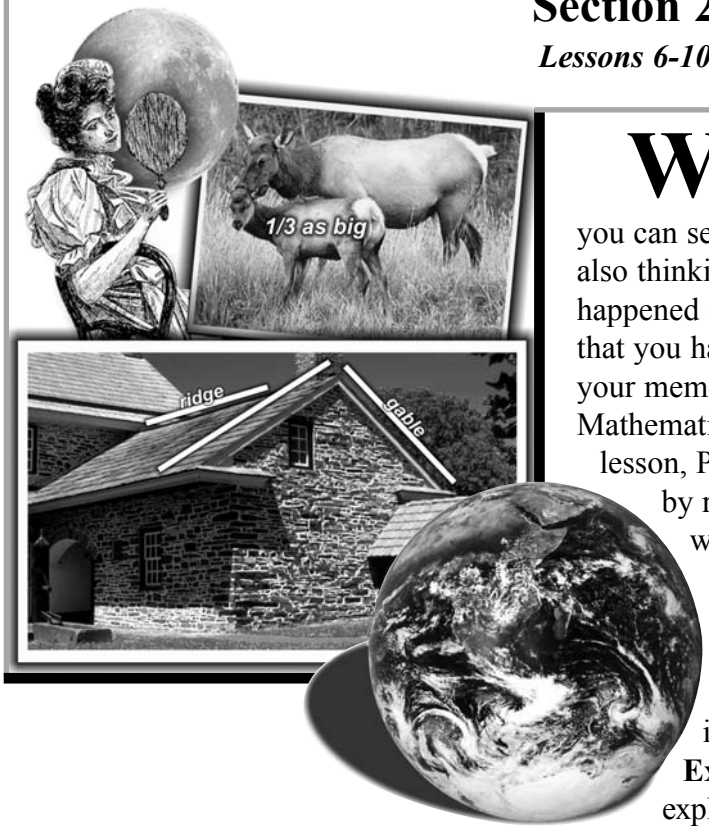
Lesson 6

Chapter 2

Section 2

Lessons 6-10

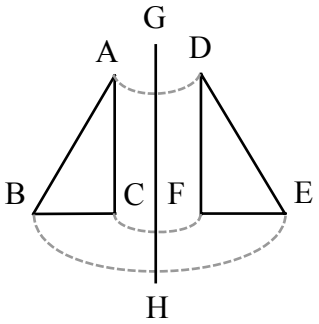




When you look in a mirror or a quiet, clear pool of water on a windless day, you can see a ***reflection*** of yourself. *Reflection* is also thinking or remembering things that have happened in your past. You often ***reflect*** on things that you have studied and this helps you to retain in your memory what you have learned. Mathematicians deal with reflections also. In this lesson, Professor K will show you what is meant by reflections in mathematics and how to work with them.

Reflections in mathematics are usually made about a line. All reflections are congruent to the original shape. An example of reflections that occur in nature is the ***symmetry*** of butterflies. Look at **Example Sets 1 and 2** as Professor K explains how to determine reflections.

Example Set 1





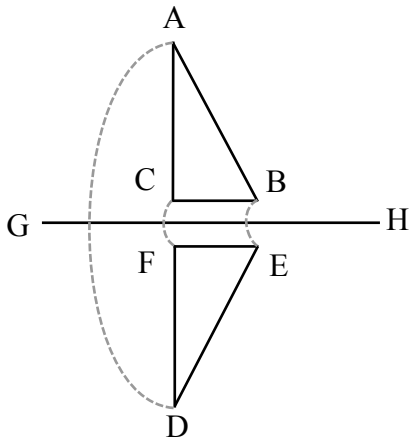
△ABC above is reflected about the vertical line GH. △DEF is the reflection of △ABC. Professor K wants you to notice that △ABC ≅ △DEF. △DEF is just like △ABC only flipped around or backward, just like when you look in a mirror your image is reversed. You should also notice that the reflected image is the same distance from the line as the original image.

VOCABULARY

Reflection: a reflected image
Reflect: to bend light; or mentally, to think or concentrate on
Symmetry: having the identical shape or pattern as a mirrored image

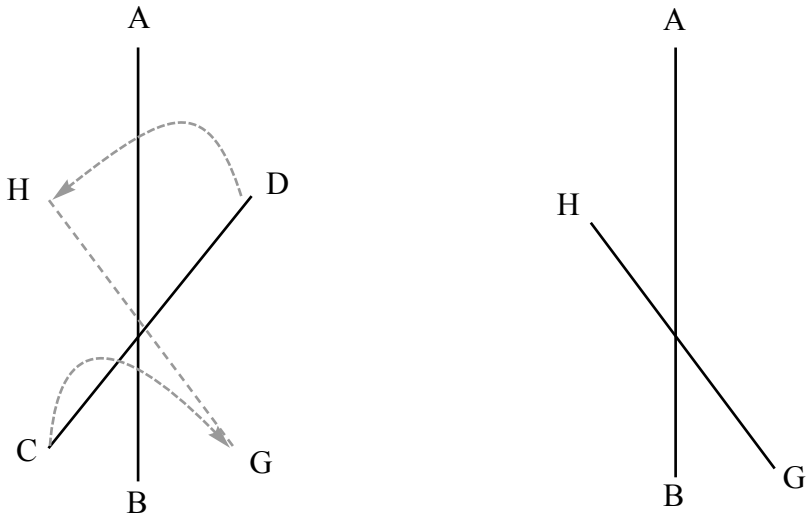
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Example Set 2



∆DEF above is a reflection about a horizontal line of ∆ABC. Notice that ∆DEF appears to be the result of flipping the image of ∆ABC around the line GH like turning a page in a notebook.

Example Set 3



In this example the line segment CD is reflected about the vertical line AB as shown. The reflected image becomes the line segment HG in the figure on the right.

Show Professor K that you understand reflections by working the activities for this lesson.

Life Principle



"Your actions are a reflection of your character."

—Anonymous

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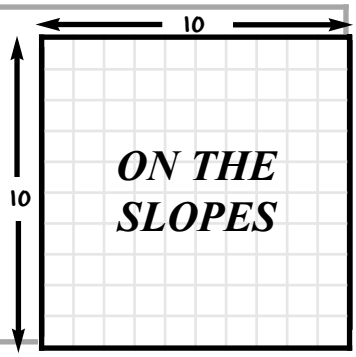
Lesson 7

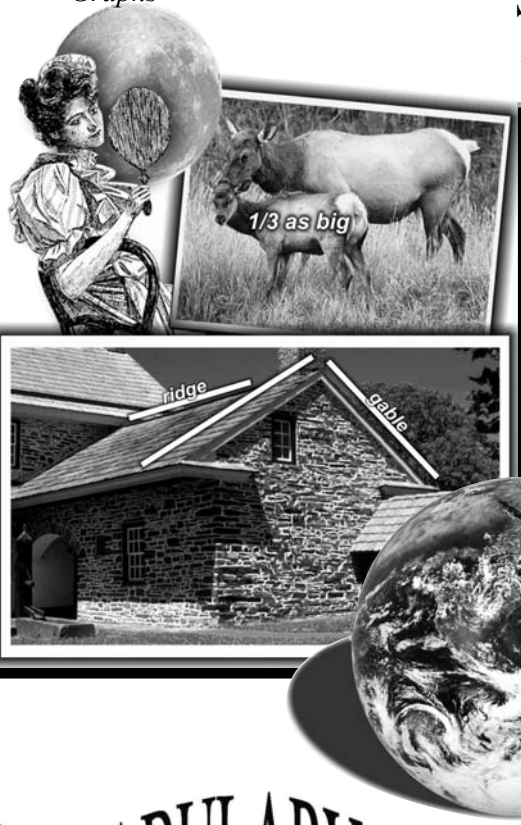
Graphs

Chapter 2

Section 2

Lessons 6-10





When building houses and other facilities, the roofs are usually slanted. Construction workers refer to this slant as *pitch*. Mathematicians refer to this slant as ***slope***. *Pitch* tells a builder what a roof will look like or how steep it will be when the building is completed. *Slope* tells a mathematician how a line appears on a graph.



Professor K

Professor K will demonstrate in this lesson how to determine the slope of a line on a graph.

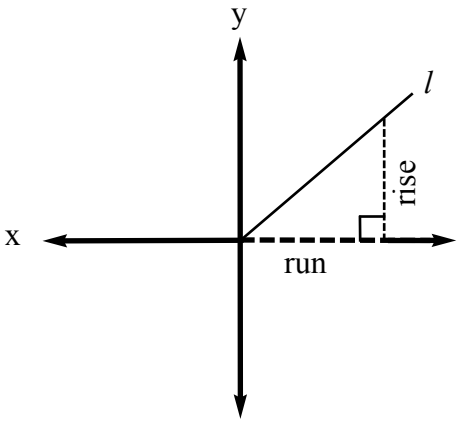


Figure 7.1

VOCABULARY

Slope, m: term used by mathematicians for the amount of steepness to a line; m =
Run: distance along the x-axis to a point
Rise: distance along the y-axis to a point
Coordinate: name of a point in a Cartesian coordinate system; (x, y)
Abscissa: the x-value of a coordinate
Ordinate: the y-value of a coordinate

Figure 7.1 shows a Cartesian coordinate system with an x-axis and a y-axis. A line, *l*, is drawn on the graph. The distance along the x-axis is known as the ***run***. The distance along the y-axis is called the ***rise***. To determine the *slope* of a line, a ratio is set up between the *rise* and the *run*.

$$slope = \frac{rise}{run}$$

The *rise* is calculated by determining the difference of the y-values of two points on a line (y_2 - y_1). The *run* is calculated by determining the difference of the x-values of two

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points on a line (x_2 - x_1). The letter *m* is used to signify the slope of a line. The equation that represents the slope of a line becomes:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Points on a Cartesian coordinate system are labeled (x, y). This is known as the ***coordinate*** of a point. The distance to the point along the x-axis is placed first in the parentheses. The x-value is known as the ***abscissa***. The distance to the point along the y-axis is placed after the comma in the parentheses. The y-value is known as the ***ordinate*** of the point.



Figure 7.2

Look at **Figure 7.2**. The coordinates of each point are listed below.

A(0, 0), B(1, 1), C(2, 2), D(3, 3), E(4, 4), F(5, 5), G(6, 6)

Look at **Example Set 1** to see how Professor K determines the slope of a line that could be drawn between each point.

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Example Set 1

Remember that slope is calculated by setting up a *ratio* between the *rise* and the *run*. The formula for the slope is:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Professor K chooses any two coordinates from the graph. He picks (0, 0) and (3, 3). Professor K lets $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (3, 3)$. Therefore, by substituting these values into the formula for slope, Professor K can calculate the slope of the line between the points (0, 0) and (3, 3).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{3-0}{3-0} = \frac{3}{3} = 1$$

Professor K calculates the slope between (2, 2) and (4, 4) the same way, allowing $(x_1, y_1) = (2, 2)$ and $(x_2, y_2) = (4, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{4-2}{4-2} = \frac{2}{2} = 1$$

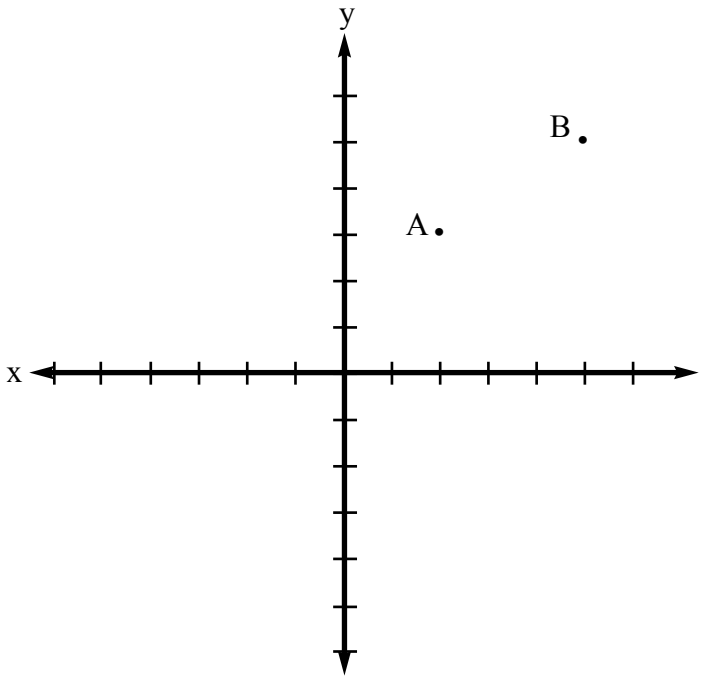
Calculations of the slope of a line between any combinations of the points on this graph would result in a slope of one, since all the points lie on a straight line.

PRINCIPLE OF SLOPE

The slope of a line between any two points on that line is equal to the slope between any two other points on the same line.



Example Set 2



Look at the graph above. Determine the slope of a line drawn between the two points.

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Step 1 Professor K determines that the coordinates of the points are (2, 3) and (5, 5).

Step 2 Professor K lets $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (5, 5)$ and substitutes these values into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{5-3}{5-2} = \frac{2}{3}$$

The slope of the line defined by the points (2, 3) and (5, 5) is the ratio $^2/_3$.

Show Professor K that you understand the slope formula by working the activities for this lesson.

Life Principle



"The path to success is sometimes a steep climb."

—Anonymous

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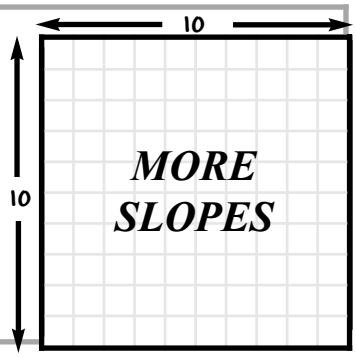
Lesson 8

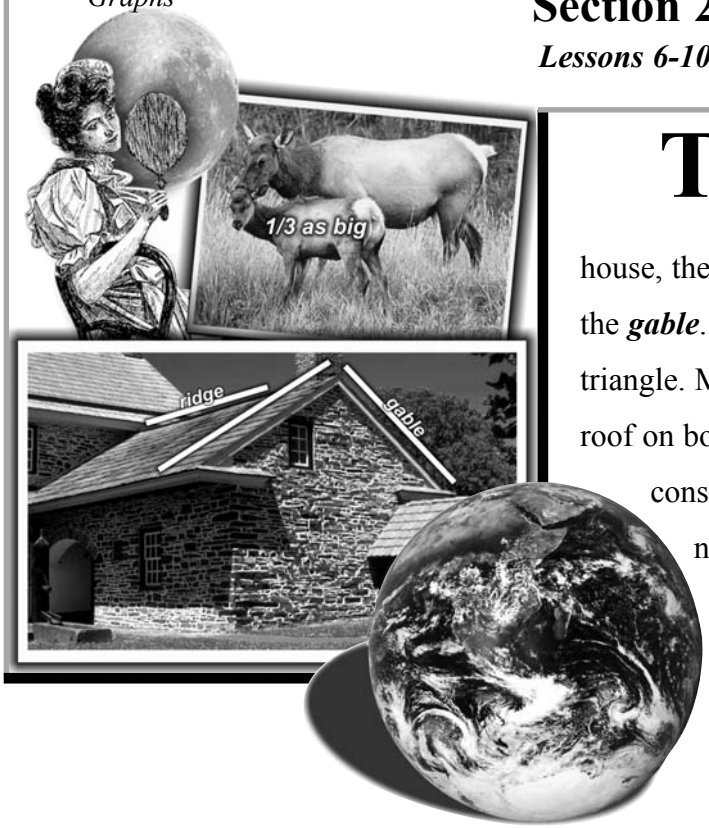
Graphs

Chapter 2

Section 2

Lessons 6-10





The roofs of houses have a ***ridge*** along the top. When looking at the end of a house, the area above the top of the wall is called the ***gable***. Most gables are in the shape of a triangle. Most of the time the slope (pitch) of the roof on both sides of the ridge is the same. When considering the slope, however, it can be negative or positive. Look at **Figure 8.1** as Professor K explains positive and negative slopes.

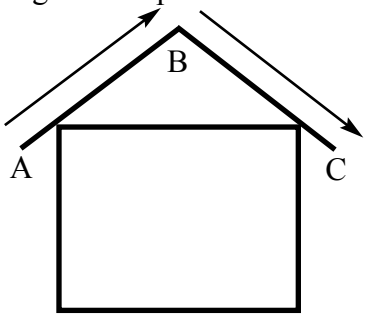


Figure 8.1

VOCABULARY

Ridge: part of a roof where the two sides of the roof meet
Gable: triangular shape of the end of a roof that is above the wall of a building
Origin: the beginning of something; in mathematics the point at which the x and y axis intersect
Rafters: timbers for supporting a roof

Figure 8.1 shows the drawing of the end of a house. Notice that the line segment AB goes in an upward direction from A to B or left to right like reading a page. The line AB has a positive slope. The line segment BC goes in a downward direction from B to C or left to right like reading a page. The Line BC has a negative slope.

Figure 8.2 shows the line segments AB and BC placed on a Cartesian coordinate system with an x-axis and a y-axis.

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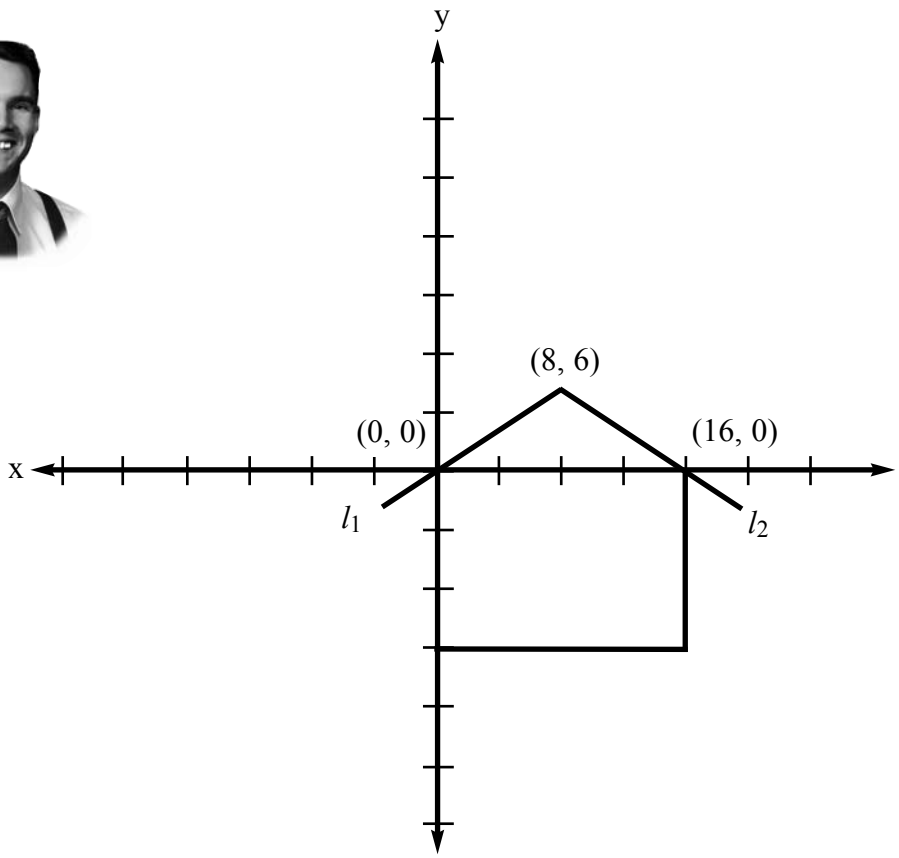


Figure 8.2

As you look at **Figure 8.2**, Professor K wants to remind you that the point at which the x-axis and y-axis intersect is known as the ***origin*** of the graph. The origin has a coordinate of (0, 0). Notice that the house was placed on the graph so that the ceiling and the top of one wall meet at the origin. This lines up the gable so that a proper calculation can be made on the slope and length of the ***rafters***. The length of the overhang would have to be added to the calculation of the length of the rafters. The length of the rafters can be calculated using the Pythagorean Theorem.

This lesson deals with slope of a line so Professor K will now calculate the slopes on the roof of the house in **Example Set 1**.

Example Set 1

The problem of calculating the first slope of the roof in **Figure 8.2** can be stated as follows:

Calculate the slope of a line through the points (0, 0) and (8, 6).

Professor K uses the slope formula to make the calculation.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{8-0} = \frac{3}{4}$$

The problem of calculating the second slope of the roof in **Figure 8.2** can be stated as follows:

Calculate the slope of a line through the points (8, 6) and (16, 0). (Remember the ordinate of any point located on the x-axis is zero. The abscissa of any point located on the y-axis is zero.)

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Professor K uses the slope formula to make the calculation.

$$m = \frac{y_2 - y_2}{x_2 - x_1} = \frac{0-6}{16-8} = \frac{-6}{8} = -\frac{3}{4}$$

Notice that the magnitudes of the slopes are the same in this case (most house roofs are symmetrical), however the signs are opposite. Line, l_1 has a positive slope and it slopes upward when moving left to right. Line, l_2 has a negative slope and slopes downward when moving from left to right.

Professor K reminds you that when subtracting a larger number from a smaller number as in the second part of **Example Set 1**, 0 - 6, you must think of the problem as subtracting the smaller number from the larger and then use the sign of the larger. Therefore, the 0 - 6 → 6 - 0 = 6 and then place a negative sign in front of the answer, -6.



RULE

All lines with a positive slope move upward going left to right.
All lines with a negative slope move downward going left to right.



Example Set 2

Calculate the slope of a line through the following points. Then tell if the graph of the line will slope upward or downward going left to right.

a) (2, 5) and (4, 6)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-5}{4-2} = \frac{1}{2}$$

The graph of the line would slope *upward* because it has a *positive slope*.

b) (1, 9) and (4, 7)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-9}{4-1} = \frac{-2}{3} = -\frac{2}{3}$$

The graph of the line would slope *downward* because it has a *negative slope.*

Show Professor K that you understand how to work with slopes by working the activities for this lesson.

Life Principle



"A positive attitude can take you upward to new heights."

—Anonymous

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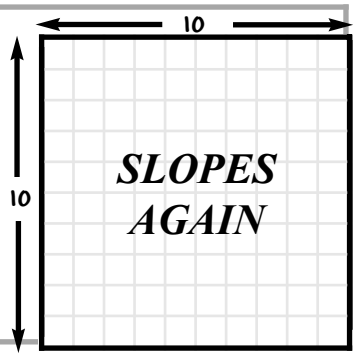
Lesson 9

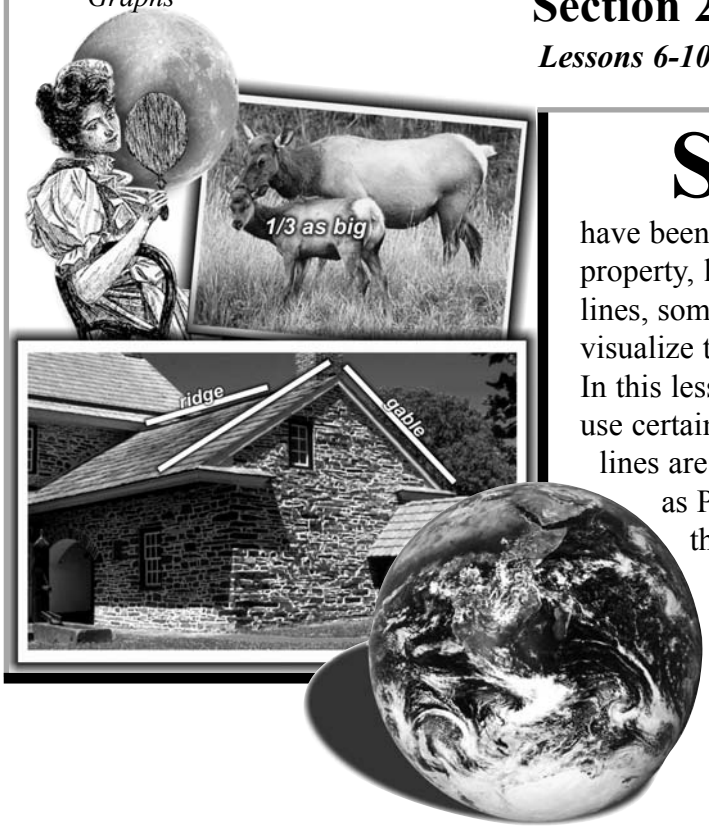
Graphs

Chapter 2

Section 2

Lessons 6-10





Since the beginning of time, rules for how people should relate to each other have been in place. These rules involve money, property, life and liberty. In working with slopes of lines, some rules can be utilized to understand and visualize the relationship of two lines to each other. In this lesson, Professor K will show you how to use certain *rules of slopes* to determine how two lines are related to each other. Look at **Figure 9.1** as Professor K begins to show you some of these relationships.

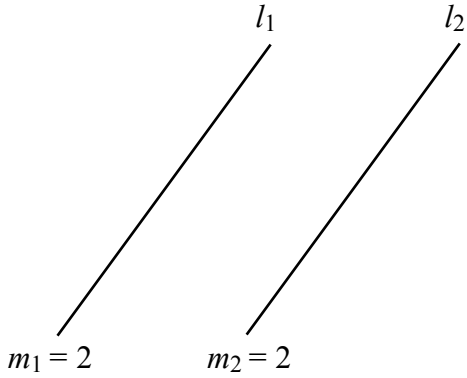


Figure 9.1

Figure 9.1 shows the drawing of two lines, l_1 and l_2. The slopes, m_1 and m_2 are the same and equal 2. Notice that the lines will never ***intersect***. Therefore, the lines are ***parallel***.

Look at **Figure 9.2** as Professor K shows you another pair of lines and describes the relationship between them.

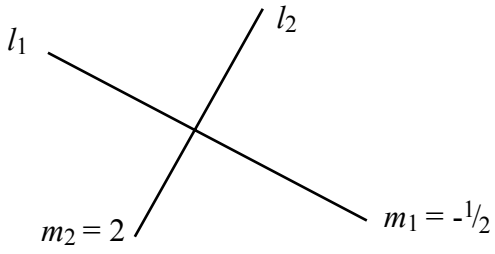


Figure 9.2

VOCABULARY

Intersect: to cross
Parallel: lines that contain no common point; lines that never intersect
Perpendicular: lines that intersect at right angles (90°)
Reciprocal: two fractions are reciprocals if when multiplied together the product is one; "flipping a number to its opposite"; $^1/_3$ and $^3/_1$ are reciprocals

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In **Figure 9.2** the lines are ***perpendicular*** (intersect at right angles). The slopes are negative ***reciprocals*** of each other. In other words the slopes have opposite signs and when the magnitudes (value without a sign) are multiplied together the result is *one*. $2 \bullet {}^{1}/_{2} = {}^{2}/_{1} \bullet {}^{1}/_{2} = {}^{2}/_{2} = 1$.

Look at **Figure 9.3** to see another relationship between lines and their slopes.

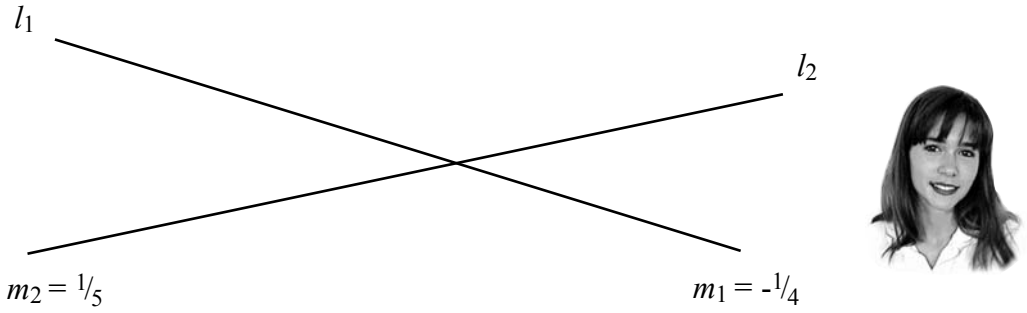


Figure 9.3

As you look at **Figure 9.3**, Professor K wants you to notice the direction that the lines slant in relationship to the value of their slopes. Notice that the slopes are not the same, therefore, the lines intersect at some point.

Figures 9.1, 9.2, and **9.3** illustrate properties 1, 2 and 3 below.

PROPERTIES OF SLOPES OF LINES

1. Parallel lines have the same slope. (**Figure 9.1**)
2. Lines with different slopes will intersect at some point. (**Figure 9.3** and **Figure 9.2**)
3. Lines with slopes that are negative reciprocals are perpendicular to each other.
4. Vertical lines have an undefined slope. (The difference of the abscissas is zero; therefore, the denominator of the slope ratio is zero. This is mathematically an undefined expression. ${}^{4}/_{0}$ = undefined)
5. Horizontal lines have a zero slope. (The difference of the ordinates is zero; therefore, the numerator of the slope ratio is zero. Any fraction with a defined denominator that also has a numerator with a value of zero has a value of zero. ${}^{0}/_{4} = 0$)

Professor K uses these properties in the following examples.

Example Set 1

Determine if the lines with the slopes described below *intersect*, are *parallel* or are *perpendicular.*

a) $m_1 = 2$ and $m_2 = 4$

a) Slopes are not equal, therefore the lines intersect (Property 2).

b) $m_1 = 3$ and m_2 = undefined

b) Slopes are not equal, therefore the lines intersect (Property 2).

c) $m_1 = 0$ and $m_2 = 1$

c) Slopes are not equal, therefore the lines intersect (Property 2).

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d) m_1 = undefined and $m_2 = 0$

d) Slopes are not equal, therefore the lines intersect (Property 2). Horizontal and vertical lines intersect at right angles. Also, the slopes are negative reciprocals (Property 3).

e) $m_1 = 6$ and $m_2 = {}^1/_6$

e) Slopes are not equal, therefore the lines intersect (Property 2).

f) $m_1 = 7$ and $m_2 = -{}^1/_7$

f) Slopes are not equal, therefore the lines intersect (Property 2). Also, the slopes are negative reciprocals, therefore the lines are perpendicular (Property 3).

In e) Professor K wants you to notice that even though the slopes are reciprocals, they are not negative reciprocals.

Example Set 2

Calculate the slope of a line through the following points. Then tell if the lines are *parallel, perpendicular* or if they *intersect.*

a) l_1 through the points (1, 2) and (3, 4)

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{3-1} = \frac{2}{2} = 1$$

b) l_2 through the points (1, 5) and (0, 3)

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{0-1} = \frac{-2}{-1} = 2$$

The lines l_1 and l_2 would *intersect* because the slopes are not the same.

Show Professor K that you understand the properties of lines presented in this lesson by working the activities for this lesson.

LIFE PRINCIPLE



"A good attitude is the reciprocal of a bad one."

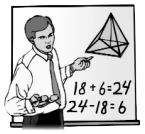
—Anonymous

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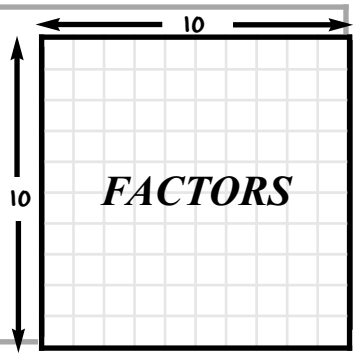
Intermediate Math Skills



Lesson 10

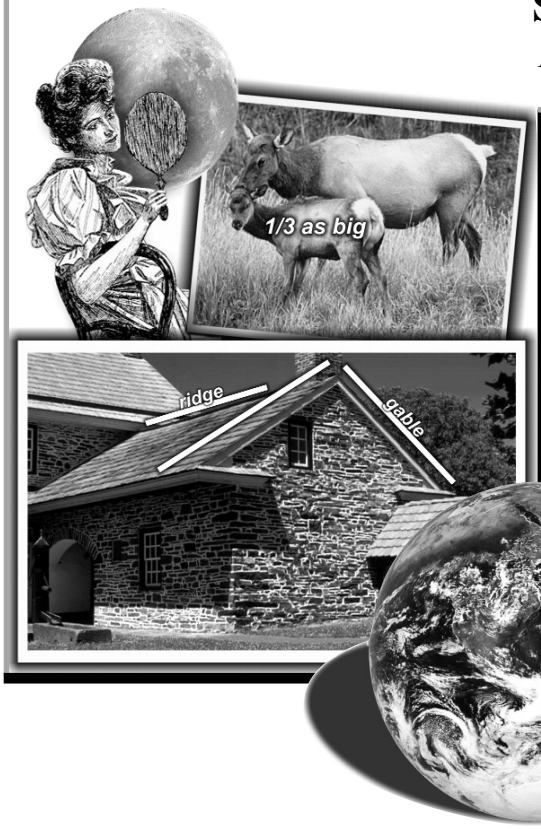
Fundamentals

Chapter 2



Section 2

Lessons 6-10



Often in mathematics, numbers are multiplied or changed by a certain ***factor***. Every day you hear someone say that something is twice as large as something else. You may also hear that one thing is one-third as much as something else. These statements are referring to numbers changed by certain factors.



Professor K

In this lesson, Professor K will present problems involving comparisons of one thing to another in relationship to a factor. Professor K will show you how to calculate the final result when these relationships are compared.

Example Set 1

The gravitational pull of the Moon is only one-sixth that of the Earth. Calculate the weight of an astronaut when he steps onto the Moon, if he weighs 240 pounds on the Earth. The factorial comparison of the astronaut's weight on the Moon is one-sixth of his 240 pound weight on Earth. Therefore, Professor K sets up the following expression to represent the weight of the astronaut on the Moon. Two hundred forty is factored to obtain 40x6. The sixes are then cancelled, leaving an answer of 40.

$$240 \text{ pounds} \times \frac{1}{6} = \frac{240 \text{ pounds}}{6} = \frac{40 \times \cancel{6} \text{ pounds}}{\cancel{6}} = 40 \text{ pounds}$$

The astronaut's weight on the Moon would be 40 pounds.

Many puzzle problems involve calculating the ages of different people compared to the ages of others. Professor K works such a problem in the next example.

VOCABULARY

Factor: something that causes a change; in mathematics one of the numbers multiplied together in a multiplication problem

Deduction: a conclusion reached by reasoning through known or inferred data

Siblings: brothers and sisters

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Example Set 2

Juan is *twice as old* as his brother, Pedro. Pedro is *one-half as old* as his sister, Adella. Adella is eight years old. Calculate the ages of Juan and Pedro. Discuss the possible conclusion that can be made about Juan and Adella after making the calculation.

Step 1 Professor K knows the actual age of Adella, 8. Therefore, he begins to work backward to find the age of Pedro. Pedro is *one-half* the age of Adella, therefore, Pedro's age is represented by the expression, $8 \bullet {}^{1}/_{2} = {}^{8}/_{2} = 4$. Therefore, Pedro is four years old.

Step 2 Juan is twice is old as Pedro, therefore, Juan's age is represented by the expression, $4 \bullet 2 = 8$. Therefore, Juan is eight years old.

Step 3 Professor K considers the three children, the information given and their ages. The first ***deduction*** that can be made about the children is that they are biological ***siblings***. The next deduction that Professor K makes is that Juan and Adella must be twins, since they are siblings and they are the same age.

Show Professor K that you understand how to work factor problems by working the activities for this lesson.

Life Principle



"Love is the factor that keeps a marriage together."

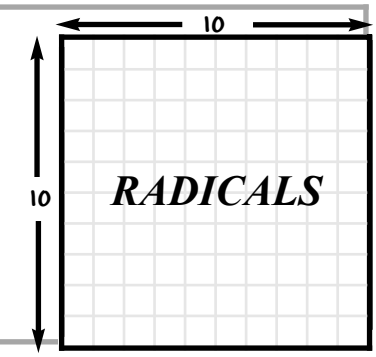
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Lesson 11

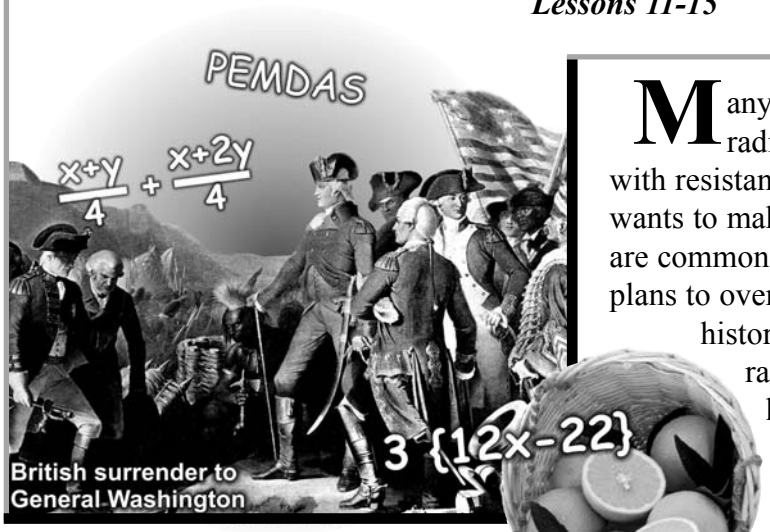
Radicals

Chapter 2



Section 3

Lessons 11-15



Many people in the world are known as radicals. To be *radical* commonly meets with resistance from others because the radical wants to make drastic changes in the way things are commonly done. To be radical may mean plans to overthrow a government. ***Reformers*** in history have been known to propose radical ideas that became accepted later by everyone. The notion of the Colonies becoming independent from England was a radical idea to most colonists.

In this lesson, Professor K will present problems involving *radicals*. Professor K reminds you that a radical expression is a mathematical expression containing a radical. Look at **Example Set 1** as Professor K gives you a review of radicals.

VOCABULARY

Reformers: people who try to make good changes in the way things are done for the betterment of all concerned
Factor: in mathematics one of numbers multiplied together in a multiplication problem
Index: the small raised number in front of a radical sign
Root: one of the equal factors of the number under the radical sign

Example Set 1

a) $\sqrt{9} = 3$	a) Three is the answer because $3 \bullet 3 = 9$.
b) $\sqrt{81} = 9$	b) Nine is the answer because $9 \bullet 9 = 81$.
c) $\sqrt{25} = 5$	c) Five is the answer because $5 \bullet 5 = 25$.
d) $\sqrt[3]{125} = 5$	d) Five is the answer because $5 \bullet 5 \bullet 5 = 125$.
e) $\sqrt[3]{-27} = -3$	e) Negative three is the answer because $(-3)(-3)(-3) = -27$.
f) $\sqrt[4]{81} = 3$	f) Three is the answer because $3 \bullet 3 \bullet 3 \bullet 3 = 81$.

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In **Example Set 1** you must remember that the radical symbol, $\sqrt{\ }$, means to calculate the two equal ***factors*** of the number under the radical sign. The symbol $\sqrt[3]{\ }$ means to find the three equal factors of the number under the radical sign. The symbol $\sqrt[4]{\ }$ means to find the four equal factors of the number under the radical symbol. The small raised number in front of the radical sign is called the ***index***. One of the equal factors of the number under the radical sign is called the ***root***. A radical without an index automatically means the root has only two factors.

Example Set 2

Write the index for each of the following radical expressions.

a) $\sqrt{4}$	a) index = 2
b) $\sqrt[4]{16}$	b) index = 4
c) $\sqrt[5]{243}$	c) index = 5
d) $\sqrt{81}$	d) index = 2



DEFINITION: PRINCIPAL SQUARE ROOT

A principal square root is the positive root in a square root problem.



The square root of a number is one of two equal factors of that number. For example, the square root of 9 could be 3 or -3, because 3 • 3 = 9 and -3 • -3 = 9. So *3* and *-3* are both square roots of *9*. However, 3 is considered the principal square root of 9 since it is the positive root. Look at **Example Set 3** as Professor K determines the principal square roots of the numbers shown.

Example Set 3

Determine the principal square root for each of the following radical expressions.

a) $\sqrt{4}$	a) roots = 2, -2; principal root = 2
b) $\sqrt{16}$	b) roots = 4, -4; principal root = 4
c) $\sqrt{25}$	c) roots = 5, -5; principal root = 5
d) $\sqrt{81}$	d) roots = 9, -9; principal root = 9

Show Professor K that you understand how to determine the roots of radicals by working the activities for this lesson.

Life Principle



"Changes come from radical thoughts."

—Anonymous

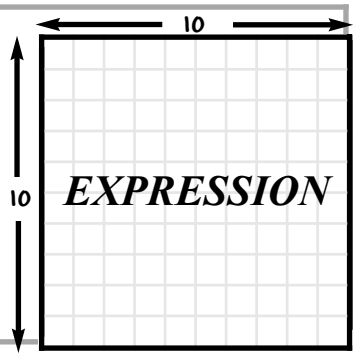
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Lesson 12

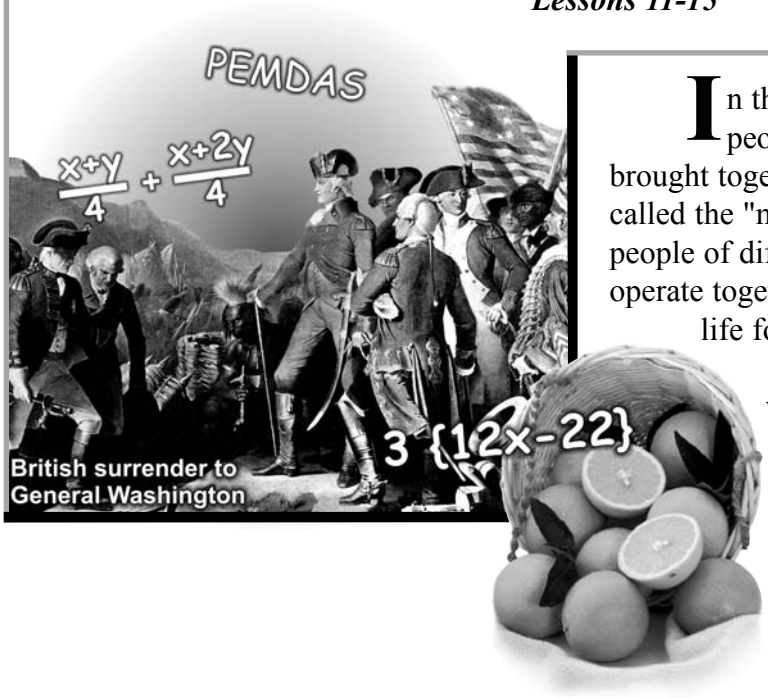
Algebra

Chapter 2



Section 3

Lessons 11-15



In the United States of America, many people from different cultures are brought together. The United States has been called the "melting pot of the world." All the people of different backgrounds and heritages operate together for freedom and a better way of life for themselves and their children.

In mathematics, ***constants*** and ***variables*** are brought together with different mathematical operations to form algebraic expressions. In this lesson, Professor K will present problems involving *algebraic expressions*. Professor K will review with you some of the terminology of algebraic expressions and introduce you to some new concepts.

Formulas are a type of algebraic expression because they contain variables. Professor K has already shown you how to evaluate formulas such as area of a rectangle, $A = l \times w$ and the volume of a rectangular prism, $V = l \times w \times h$. Look at **Example Set 1** as Professor K gives you a review of evaluating these formulas.

VOCABULARY

Constant: a number or letter whose value does not change
Variable: a letter used to represent a number
Coefficient: the number in front of a variable in a term
Term: a number, variable or a product of more than one number, or variable not separated by addition or subtraction symbols

Example Set 1

a) Calculate the area of a rectangle with a length of 9 meters and a width of 7 meters. Substitute the values of the length and width into the formula for the area of a rectangle.

$A = l \times w = 9 \text{ m} \times 7 \text{ m} = 63$ square meters

b) Calculate the volume of a cube with an edge of 4 inches. Substitute the values of the length of an edge of the cube into the formula for the volume of a cube (a type of rectangular prism)

$V = l \times w \times h = 4" \times 4" \times 4" =$

16 square inches × 4 inches = 64 cubic inches

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Remember that the Associative Property of Multiplication allows you to group factors together for multiplication in steps as in **Example Set 1 b)**.

Associative Property of Multiplication

The factors in a mathematical expression can be grouped in any way.

- $3 \bullet 4 \bullet 5 = (3 \bullet 4) \bullet 5 = 3 \bullet (4 \bullet 5)$
- $x \bullet y \bullet z = (x \bullet y) \bullet z = x \bullet (y \bullet z)$

Earlier, Professor K explained that evaluating some mathematical expressions was like going to the produce section of a grocery store. Apples and oranges are grouped separately, not together. This grouping of like objects in a mathematical expression is known as *combining like terms*. Professor K reviews this concept for you in **Example Set 2**.

Example Set 2

Determine the ***coefficient*** of each of the following ***terms.***

a) $3x$ . . . . . coefficient 3
b) $4a$ . . . . . coefficient 4
c) $35y$ . . . . . coefficient 35
d) x . . . . . coefficient 1 (when no number or constant is shown it is understood to be 1)

Definition: Like Terms

Like terms are terms that contain the exact same variable or variables.



- $3x$, $56x$, and $20x$ are *like terms*
- 4, 6, 90, 34 are *like terms*
- $4xy$, $39xy$, $25xy$, and xy are *like terms*

Definition: Unlike Terms

Unlike terms are terms that contain different variables.

- x, x^2 and x^3 are *unlike terms*

Look at **Example Set 3** as Professor K reviews combining like terms in mathematical expressions.

Example Set 3

Simplify the mathematical expressions below (combine all *like terms*).

a) $3x + 4 + 2x + 5$
$(3x + 2x) + (4 + 5)$
$x(3+2) + (4 + 5)$
$5x + 9$

a) The Commutative Property allows you to move *like terms* together. The Distributive Property allows you to divide out the x. Add all the constants.

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b) $xy + 3xyz + 4xy + 4 + 5xyz$
$(xy + 4xy) + (3xyz + 5xyz) + 4$
$xy(1 + 4) + xyz(3 + 5) + 4$
$5xy + 8xyz + 4$

b) The Commutative Property allows you to move *like terms* together. The Distributive Property allows you to divide out the x. Add all the constants.

c) $4x + 2x^2 + 3x^3 + 3x^2 + 10x + 5x^3$
$(4x + 10x) + (2x^2 + 3x^2) + (3x^3 + 5x^3)$
$x(4 + 10) + x^2(2 + 3) + x^3(3 + 5)$
$14x + 5x^2 + 8x^3$

c) The Commutative Property allows you to move *like terms* together. Remember x, x^2 and x^3 are not the same variable, therefore they are not *like terms*. The Distributive Property allows you to divide out the x. Add all the constants.

Commutative Property of Addition

The Commutative Property of Addition states that the order in which numbers are added together does not matter.

- $4 + 3 = 3 + 4 = 7$
- $23 + 17 = 17 + 23 = 40$
- $2 + 5 + 6 = 6 + 5 + 2 = 2 + 6 + 5 = 6 + 2 + 5 = 5 + 6 + 2 = 5 + 2 + 6 = 13$

Distributive Property of Multiplication

The Distributive Property of Multiplication across addition and subtraction in a mathematical expression is shown in generalized form below.

Addition: $a[x + y] = a \bullet x + a \bullet y$
Subtraction: $a[x - y] = a \bullet x - a \bullet y$

- $3(x + 4) = (3 \bullet x) + (3 \bullet 4) = 3x + 12$
- $3(x - 4) = (3 \bullet x) - (3 \bullet 4) = 3x - 12$

Show Professor K that you understand how to simplify mathematical expressions by working the activities for this lesson.

Life Principle



"Freedom requires diligent effort."
—Anonymous

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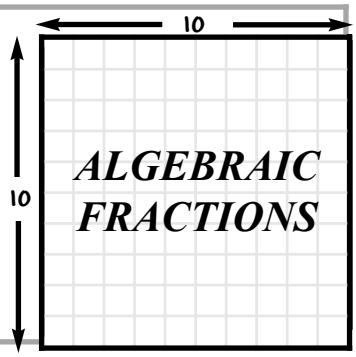
Intermediate Math Skills



Fractions

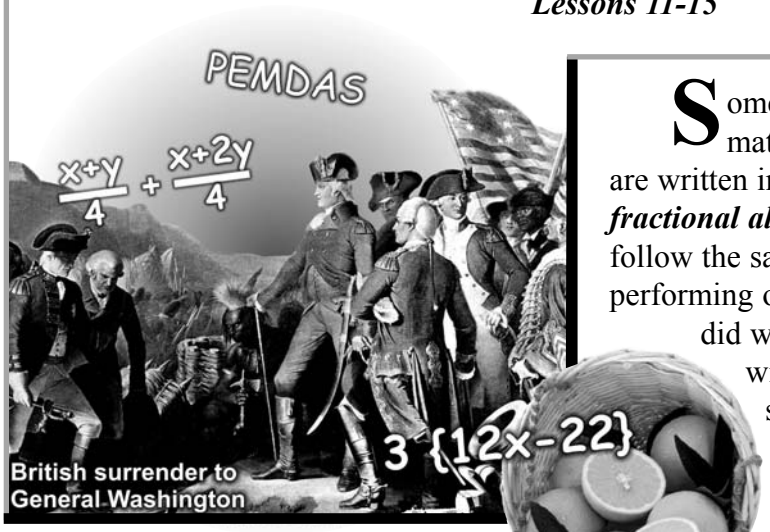
Lesson 13

Chapter 2



Section 3

Lessons 11-15



Some algebraic expressions in mathematics contain fractions, or they are written in fraction form. To simplify ***fractional algebraic*** expressions, you must follow the same mathematical rules when performing operations on the fractions as you did with numerical fractions (fractions with only numbers). However, the simplification of fractional algebraic expressions does have a few special rules. Professor K will review the basic operations of fractions as he includes algebraic expressions in his discussion.

Refer to the rules below as you look at the examples in this lesson. Professor K reviews numerical fractions following the rules listed below in **Example Set 1**.

Rules for operations on fractions

1. To add fractions: the denominators must be equal, then add the numerators.
2. To subtract fractions: the denominators must be equal, then subtract the numerators.
3. To multiply fractions: place the product of the factors of the numerators in the numerator of the answer and place the product of the factors of the denominators in the denominator of the answer. (The denominators do not have to be the same.)
4. To divide fractions: find the reciprocal (flip over) of the fraction to the right of the division operation symbol, then multiply the resulting fractions according to Rule #3 (above).



Fractional: part of; having to do with fractions
Algebraic: having to do with algebra; usually involves terms with variables

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Example Set 1

Perform the indicated operations and reduce the answer.

a) $\frac{2}{5} \bullet \frac{3}{4} = \frac{2 \bullet 3}{5 \bullet 4} = \frac{6}{20} = \frac{\cancel{2} \bullet 3}{\cancel{2} \bullet 10} = \frac{3}{10}$

a) When multiplying fractions, the denominators are not required to be the same. Multiply the numerators and place the product in the numerator of the answer. Do the same with the denominators. Reduce the fraction.

b) $\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \frac{7}{5} = \frac{5}{5} + \frac{2}{5} = 1\frac{2}{5}$

b) When adding fractions, the denominators must be the same before you add the numerators. The denominator stays the same. If an improper fraction results, change it to a mixed number.

c) $\frac{6}{7} - \frac{4}{7} = \frac{6-4}{7} = \frac{2}{7}$

c) When subtracting fractions, the denominators must be the same; then you can subtract the numerators, while keeping the denominator the same.

d) $\frac{3}{8} \div \frac{3}{32} = \frac{3}{8} \bullet \frac{32}{3} = \frac{\cancel{3} \bullet 32}{8 \bullet \cancel{3}} = \frac{32}{8} = \frac{\cancel{8} \bullet 4}{\cancel{8}} = \frac{4}{1} = 4$

d) When dividing two fractions, you must find the reciprocal of the fraction to the right of the division operation symbol. In other words, flip upside down the fraction to the right. Then multiply the numerators and denominators according to Rule #3. Reduce the answer if possible.

In **Example Set 2** Professor K shows you how to simplify algebraic expressions containing variables. Remember to refer to the rules for operations on fractions as you look at **Example Set 2**.



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Example Set 2

Perform the indicated operations and reduce the answer.

a) $\frac{a}{5} \bullet \frac{b}{4} = \frac{a \bullet b}{5 \bullet 4} = \frac{ab}{20}$

a) When multiplying fractions the denominators are not required to be the same. Multiply the numerators and place the product in the numerator of the answer. Do the same with the denominators. Reduce the fraction.

b) $\frac{x}{5} + \frac{y}{5} = \frac{x+y}{5}$

b) When adding fractions, the denominators must be the same; then add the numerators while keeping the denominator the same. Notice that the *unlike terms* in the numerator cannot be combined.

c) $\frac{5}{w} - \frac{4}{w} = \frac{5-4}{w} = \frac{1}{w}$

c) When subtracting fractions, the denominators must be the same; then subtract the numerators while keeping the denominator the same. Notice that the numerator contains no variables so the *like terms* can be added together.

d) $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{a \bullet d}{b \bullet c} = \frac{ad}{bc}$

d) When dividing two fractions, you must find the reciprocal of the fraction to the right of the division operation symbol. In other words, flip upside down the fraction to the right. Then multiply the numerators and denominators according to Rule #3. Reduce the answer if possible.

Look at **Example Set 3** as Professor K reviews combining *like terms* in fractional expressions.

Example Set 3

Perform the indicated operations in the mathematical expressions below. Be sure to reduce the answer and/or simplify (combine all like terms) when possible.

a) $\frac{x+y}{4} + \frac{x+2y}{4} =$

$\frac{x+y+x+2y}{4} =$

$\frac{(x+x)+(y+2y)}{4} =$

$\frac{2x+3y}{4}$

a) Since the denominators are the same, *4*, you can add the numerators. The Commutative Property then allows you to move *like terms* together. Add all the coefficients. Remember you cannot combine *unlike terms*.



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b) $\frac{4x+3y}{x} - \frac{2x+2y}{x} =$

$\frac{4x+3y-(2x+2y)}{x} =$

$\frac{4x+3y-2x-2y}{x} =$

$\frac{(4x-2x)+(3y-2y)}{x} =$

$\frac{2x+y}{x}$

b) Since the denominators are the same (x), you can subtract the numerators. Use the Distributive Property to distribute the negative sign within the parentheses. $-(2x + 2y) = -2x - 2y$. The Commutative Property then allows you to move *like terms* together. Add all the coefficients. Remember you cannot combine *unlike terms*. It is also important to notice that you cannot divide out (cancel) the x in the numerator and denominator of the final expression. The terms in the numerator are separated by an addition symbol and each term does not contain an x.

Show Professor K that you understand how to simplify mathematical expressions by working the activities for this lesson.

Life Principle



"To maintain freedom, we must all work together."

—Anonymous

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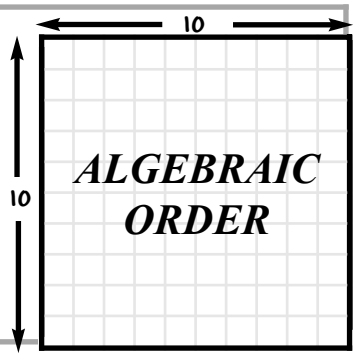
Intermediate Math Skills



Algebra

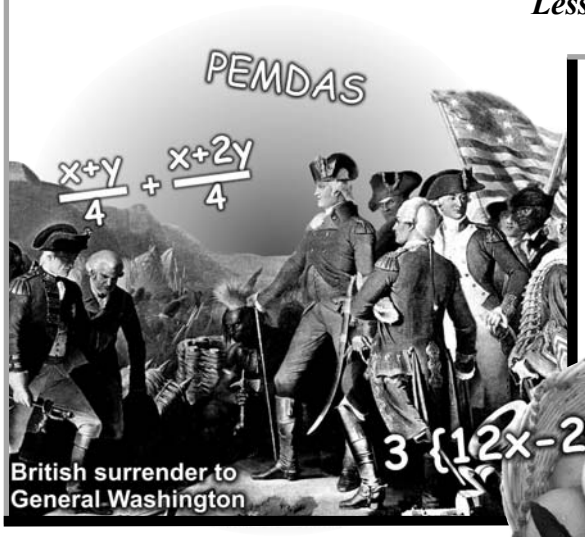
Lesson 14

Chapter 2



Section 3

Lessons 11-15





Professor K

Often, algebraic expressions contain terms that are raised to *powers*, or they are written in exponential form. They also contain addition, subtraction, multiplication and division operation symbols. To evaluate these expressions you must remember the ***acronym***, ***PEMDAS***. Professor K will review evaluating numerical expressions using PEMDAS and then show you how to use PEMDAS in algebraic expressions containing variables.

Refer to the rules of PEMDAS below as you look at the examples in this lesson. Professor K reviews numerical fractions following the rules of PEMDAS listed below in **Example Sets 1 and 2**.

RULES FOR PEMDAS (PARENTHESES, EXPONENTS, MULTIPLICATION, DIVISION, ADDITION, SUBTRACTION)
"PLEASE EXCUSE MY DEAR AUNT SALLY"

1. **Parentheses:** Perform all operations within the innermost grouping symbol first.
2. **Exponents:** Perform calculations on all terms containing exponents.
3. **Multiplication/Division:** Perform all multiplications or divisions as you come to them, left to right.
4. **Addition/Subtraction:** Perform all additions and subtractions as you come to them, left to right.

VOCABULARY

Acronym: a word formed from the first letter of each word of a group of words
PEMDAS: order of operations, Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction

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Example Set 1

Step 1	$4[5 + 6(4 - 2)] + 7$	Looking at the expression to be evaluated, two sets of grouping symbols are seen: [( )]. You must go to the innermost set of symbols ( ) first and perform the mathematical operation shown. *4 - 2 = 2*.
Step 2	$4[5 + 6(2)] + 7$	Go to the only grouping symbols remaining [ ] and perform the operations within the grouping symbols following steps two through four of the PEMDAS rules. Perform the multiplication first: *6(2) = 12*.
Step 3	$4[5 + 12] + 7$	The only operation remaining within the square brackets is addition. Perform it now. *5 + 12* = 17.
Step 4	$4[17] + 7$	Looking back at PEMDAS the *P* and *E* have been completed. The numerical expression is simplified to *4[17] + 7*. The next step going left to right in PEMDAS is *M*, multiplication of *4(17)*. 4(17) = 68.
Step 5	$68 + 7$	All that remains is a simple addition problem, *68 + 7*. Perform the operation and calculate the answer: 68 + 7 = 75.

Example Set 2

Step 1	$3^2 + [5^2 - 6(3 - 2)] + 9$	Looking at the expression to be evaluated, two sets of grouping symbols are seen: [( )]. You must go to the innermost set of symbols ( ) first and perform the mathematical operation shown according to PEMDAS. *3 - 2* = 1.
Step 2	$3^2 + [5^2 + 6(1)] + 9$	Go to the only grouping symbols remaining [ ] and perform the operations within the grouping symbols following steps two through four of the PEMDAS rules. Exponents should be first; therefore, calculate 5^2 = 25.
Step 3	$3^2 + [25 + 6(1)] + 9$	Remaining within the square brackets is *25 + 6(1)*. Perform the multiplication first: 6(1) = 6.

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Step 4	$3^2 + [25 + 6] + 9$	The only operation remaining within the square brackets is addition. Perform it now. $25 + 6 = 31$.
Step 5	$3^2 + 31 + 9$	Looking back at PEMDAS, the *P* has been completed. The numerical expression has simplified to $3^2 + 31 + 9$. The next step going left to right in PEMDAS is *E*, Exponents, therefore the value of 3^2 is calculated: $3^2 = 9$.
Step 6	$9 + 31 + 9$ $40 + 9$	All that remains is a simple addition problem, $9 + 31 + 9$. Perform the operation and calculate the answer: $40 + 9 = 49$.

The previous two examples were of simplifying or evaluating numerical expressions. In **Example Sets 3 and 4**, Professor K will show you how to simplify an algebraic expression using the Order of Operations.

Example Set 3

Simplify the following algebraic expression.

Step 1	$x + \{2(x - 3)\} + 5$	When simplifying algebraic expressions you must follow the same PEMDAS rules as you do for numeric expressions. Go to the innermost grouping symbol, ( ) and perform all the possible operations within it. In this problem, the operation in the parentheses is subtraction. After close observation, you can see that the terms are not alike: $(x - 3)$. Therefore, the terms cannot be subtracted from each other. All possible operations are accomplished, so move on to the next grouping symbol, { }. Perform all possible operations within the curly brackets. Use the Distributive Property. $2(x - 3) = 2 \bullet x - 2 \bullet 3 = 2x - 6$
Step 2	$x + 2x - 6 + 5$	$2x - 6$ is an algebraic expression consisting of two *unlike terms*. Therefore, it is simplified as much as possible.
Step 3	$(x + 2x) + (-6 + 5)$	Group *like terms* together. When the terms are grouped in parenthesis, a plus sign is placed before the parenthesis. All subtraction becomes addition of the negative sign and the number following it is put in parenthesis.

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Step 4	$x(1 + 2) + (-6 + 5)$	Divide out the x using the Distributive Property.
Step 5	$3x + (-1)$	Combine numbers within the parentheses. Remember, to add negative and positive numbers, subtract the smaller number from the larger number and use the sign of the larger number as the sign of the answer. *-6 + 5* = - (6 - 5) = - (1) = -1
Step 6	$3x$ -1	Finally, you must drop the parentheses by changing the sign in front of the one. Positive times Negative = Negative. The algebraic expression cannot be simplified any further because the two terms are not *like terms*. *3x -1* is the answer to the simplification of the original expression.

Example Set 4

Simplify the following algebraic expression.

Step 1	$x^2 + 2\{4[2(x + 3)] + x^2\} + 8$	Remember you must follow the PEMDAS rules. Go to the innermost grouping symbol, ( ) and perform all the possible operations within it. In this problem the operation in the parentheses is addition. After close observation, you can see that the terms are not alike, *(x + 3)*. Therefore, the terms cannot be added.
Step 2	$x^2 + 2\{4[2x + 6] + x^2\} + 8$	All possible operations are accomplished, so move on to the next grouping symbol, [ ]. Perform all possible operations within the square brackets. Use the Distributive Property. *2(x + 3)* = $2 \bullet x + 2 \bullet 3 = 2x + 6$. *2x + 6* is an algebraic expression consisting of two *unlike terms*. Therefore, it is simplified as much as possible.
Step 3	$x^2 + 2\{8x + 24 + x^2\} + 8$ $x^2 + 16x + 48 + 2x^2 + 8$ $x^2 + 2x^2 + 16x + 48 + 8$	Using the Distributive Property, drop the square brackets *4[2x + 6] = 8x + 24* and curly brackets. *2{8x + 24 + x²} = 16x + 48 + 2x²*.

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Step 4	$(x^2 + 2x^2) + 16x + (48 + 8)$	Group *like terms* together using the Commutative Property.
Step 5	$3x^2 + 16x + 56$	Divide out the x^2 using the Distributive Property: $x^2 + 2x^2 = x^2(1 + 2)$. Combine numbers within the parentheses. The algebraic expression cannot be simplified any further because the three terms are not *like terms*. $3x^2 + 16x + 56$ is the answer to the simplification of the original expression.

Show Professor K that you understand how to simplify mathematical expressions by working the activities for this lesson.

Life Principle



"Problems in life are simplified
when taken one step at a time."
—Anonymous

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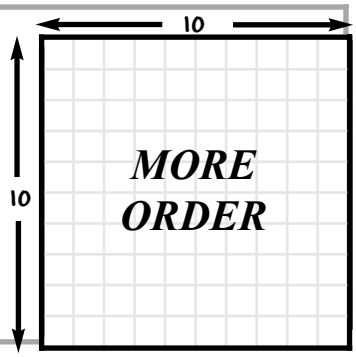
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Algebra

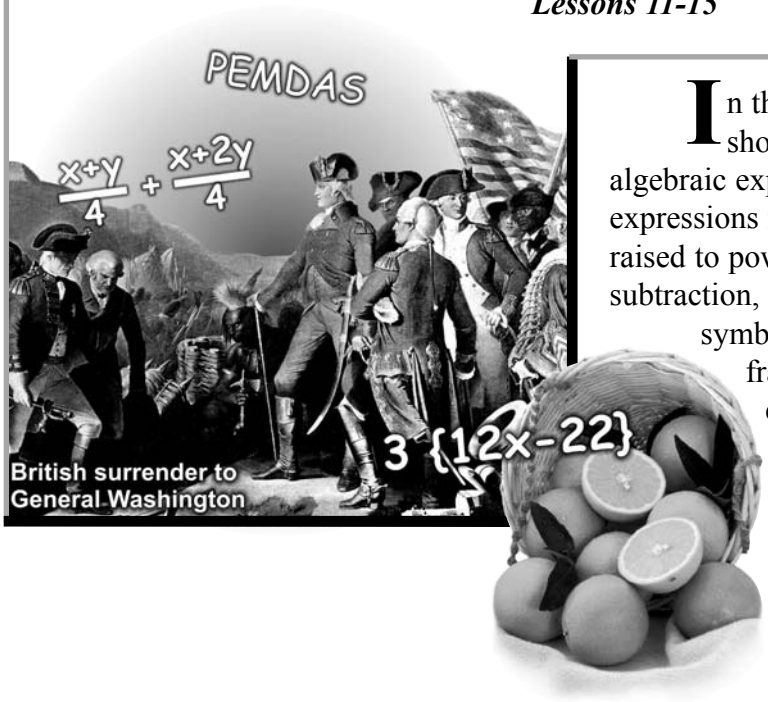
Lesson 15

Chapter 2



Section 3

Lessons 11-15



In the previous lesson, Professor K showed you how to simplify basic algebraic expressions. Often however, algebraic expressions not only contain terms that are raised to powers and contain addition, subtraction, multiplication and division operation symbols, but they also may be written in fraction form. To ***evaluate*** these expressions you must remember the acronym, PEMDAS plus an added feature of fractions. Professor K will again review evaluating numerical expressions using PEMDAS, along with the added feature, and then ***progress*** to algebraic expressions to show you how to use PEMDAS in such expressions.

Refer to the rules of PEMDAS in the previous lesson as needed as you look at the examples in this lesson. Professor K reviews numerical fractions following the rules of PEMDAS listed below in **Example Set 1**.

Example Set 1

Step 1 $\dfrac{5[3+2(4-1)]}{2[2(5-2)]}$

Looking at the expression to be evaluated, you can see that the numerical expression is written in fraction form. You must evaluate the expression in the numerator and then the denominator separately and then bring them together to completely evaluate or simplify the expression.

VOCABULARY

Evaluate: simplify; solve; determine the answer; reach a conclusion
Progress: move forward

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Step 2

$5[3 + 2(4 - 1)]$
$5[3 + 2(3)]$
$5[3 + 6]$
$5[9]$
45

Looking first at the numerator, two sets of grouping symbols are seen: [( )]. You must use PEMDAS as in the previous lesson as shown going to the innermost set of symbols ( ) first and working your way outward. The evaluation of the numerator is *45*.

Step 3

$2[2(5 - 2)]$
$2[2(3)]$
$2[6]$
12

Now you must evaluate the algebraic expression in the denominator. Once again two sets of grouping symbols are seen and PEMDAS rules are followed as before. The evaluation of the denominator is *12*.

Step 4

$$\frac{45}{12} =$$

$$\frac{\cancel{3} \bullet 15}{\cancel{3} \bullet 4} =$$

$$\frac{15}{4} = 3\frac{3}{4}$$

You now must bring the numerator and denominator together in the same fraction again. The fraction is divided up into its factors and reduced as shown to obtain $^{15}/_{4}$ or $3^{3}/_{4}$ as the final evaluation of the original expression.

The previous example showed how to simplify or evaluate a fractional numerical expression. In **Example Set 2** Professor K will show you how to simplify a fractional algebraic expression using the Order of Operations.

Example Set 2

Simplify the following algebraic expression.

Step 1

$$\frac{3\{2 + 4[x + 2(x - 3)]\}}{5(x + 7)}$$

Looking at the expression to be evaluated, you can see that the numerical expression is written in fraction form. You must evaluate the expression in the numerator and then the denominator separately and then bring them together to completely evaluate or simplify the expression.



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Step 2

$3\{2 + 4[x + 2(x - 3)]\}$
$3\{2 + 4[x + 2 \bullet x - 2 \bullet 3)]\}$
$3\{2 + 4[x + 2x - 6]\}$
$3\{2 + 4[x(1 + 2) - 6]\}$
$3\{2 + 4[3x - 6]\}$
$3\{2 + 12x - 24\}$
$3\{2 - 24 + 12x\}$
$3\{- 22 + 12x\}$
$3\{12x - 22\}$
$36x - 66$

Looking first at the numerator, three sets of grouping symbols are seen: { [( )] }. You must use PEMDAS as in the previous lesson as shown going to the innermost set of symbols ( ) first and working your way outward. Remember, if terms are not alike, you cannot combine them. The evaluation of the numerator is *36x - 66*.

Step 3

$5(x + 7)$
$5x + 35$

Now you must evaluate the algebraic expression in the denominator. Only one set of grouping symbols is seen and you must use the Distributive Property to determine the evaluation of the denominator. The evaluation of the denominator is *5x + 35*.

Step 4

$$\frac{36x - 66}{5x + 35}$$

You now must bring the numerator and denominator together in the same fraction again. The fraction is in its final form. Remember you cannot divide out the *x* because you cannot cancel across addition and/or subtraction operation symbols.

Show Professor K that you understand how to simplify mathematical expressions by working the activities for this lesson.

Life Principle



"Every problem has a solution."

—Anonymous

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Intermediate Math Skills

Mathematics Charts

LENGTH - ENGLISH SYSTEM

12 inches	=	1 foot
3 feet	=	1 yard
5.5 yards	=	1 rod
320 rods	=	1 mile
1,760 yards	=	1 mile
5,280 feet	=	1 mile

VOLUME - ENGLISH SYSTEM

1 tablespoon, Tbsp, T	=	3 teaspoons, tsp
1 cup, c	=	16 tablespoons, Tbsp, T
1 pint, pt	=	2 cups, c
1 quart, qt	=	2 pints, pts
1 gallon, gal	=	4 quarts, qts

WEIGHT - ENGLISH SYSTEM

1 pound, lb	=	16 ounces, oz
1 hundredweight, cwt	=	100 pounds, lbs
1 ton	=	20 hundredweight, cwt
1 ton	=	2000 pounds, lbs

LENGTH - METRIC SYSTEM

1 meter	=	1,000 millimeters
1 meter	=	100 centimeters
1 kilometer	=	1,000 meters
1 centimeter	=	10 millimeters

VOLUME - METRIC SYSTEM

1 liter	=	1,000 milliliters
1 liter	=	100 centiliters
1 kiloliter	=	1,000 liters
1 centiliter	=	10 milliliters

WEIGHT - METRIC SYSTEM

1 gram	=	1,000 milligrams
1 gram	=	100 centigrams
1 kilogram	=	1,000 grams
1 centigram	=	10 milligrams

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Mathematics Charts

PERFECT SQUARES

$\sqrt{1}=1$	$\sqrt{25}=5$	$\sqrt{81}=9$	$\sqrt{169}=13$
$\sqrt{4}=2$	$\sqrt{36}=6$	$\sqrt{100}=10$	$\sqrt{196}=14$
$\sqrt{9}=3$	$\sqrt{49}=7$	$\sqrt{121}=11$	$\sqrt{225}=15$
$\sqrt{16}=4$	$\sqrt{64}=8$	$\sqrt{144}=12$	$\sqrt{256}=16$

PERFECT CUBES

$\sqrt[3]{1}=1$	$\sqrt[3]{-8}=-2$	$\sqrt[3]{64}=4$	$\sqrt[3]{-125}=-5$
$\sqrt[3]{-1}=-1$	$\sqrt[3]{27}=3$	$\sqrt[3]{-64}=-4$	$\sqrt[3]{216}=6$
$\sqrt[3]{8}=2$	$\sqrt[3]{-27}=-3$	$\sqrt[3]{125}=5$	$\sqrt[3]{-216}=-6$

$$P = \frac{\text{the number of elements in the set containing favorable outcomes}}{\text{the number of elements in the sample set}}$$

$$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

FORMULAS

Surface Area	Area	Volume	Perimeter/ Circumference	Others
Sphere: $A_s = 4\pi r^2$	**Triangle:** $A = {}^1\!/_2 bh$	**Sphere:** $V = {}^4\!/_3 \pi r^3$	**Perimeter of a Rectangle:** $P = 2w + 2l$	**Pythagorean Theorem:** $c^2 = a^2 + b^2$
Cylinder: $A_s = 2\pi rh + 2\pi r^2$	**Rectangle:** $A = l \times w$	**Cone:** $V = {}^1\!/_3 \pi r^2 h$	**Circumference of a Circle:** $C = 2\pi r$ or $C = \pi d$	**Convert to Fahrenheit:** $F = {}^9\!/_5 C + 32$
Cube: $A_s = 6s^2$	**Circle:** $A = \pi r^2$	**Rectangular Prism:** $V = l \times w \times h$		**Convert to Celsius:** $C = {}^5\!/_9 (F - 32)$
Lateral Surface Area of a Cone: $A_{ls} = \pi rl$	**Trapezoid:** $A = {}^1\!/_2 (b_1 + b_2)h$	**Square Pyramid:** $V = ({}^1\!/_3) lwh$		**Distance:** $d = r \bullet t$
		Triangular Prism: $V = {}^1\!/_2 bhl$		**Slope:** $y = mx = b$
		Cylinder: $V = \pi r^2 h$		**Simple Interest:** $I = prt$

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Mathematics Charts

TIME CONVERSIONS

1 week = 7 days
1 year = 12 months

CONVERSIONS

English to Metric	
1 ounce (oz)	28.35 grams (g)
1 pound (lb)	454 grams (g)
1 ton (T)	907.2 kilograms (kg)

Metric to English	
1 gram (g)	.035 ounce (oz)
1 kilogram (kg)	2.2 pounds (lb)
1 metric ton (mt)	2200 pounds (lb)

English to Metric	
1 cubic inch	16.39 cubic cm
1 gallon	3.785 liters
1 cubic foot	28.316 liters
1 quart	.946 liters

Metric to English	
1 cubic cm	.061 cubic inch
1 cubic meter	35.315 cubic feet
1 liter	1.05 quarts
1 liter	.264 gallon

English to Metric	
1 inch	2.54 cm
1 foot	0.3048 meter
1 yard	0.9144 meter
1 mile	1.6 km

Metric to English	
1 centimeter	0.3937 in.
1 meter	39.37 in.
1 meter	1.09 yds
1 kilometer	0.6213 mile

AREA CONVERSIONS

1 square centimeter = 0.155000 sq in.
1 square meter = 10.763910 sq ft
1 square inch = 645.16 sq mm
1 square foot = 0.09290304 sq meter
1 square yard = 0.836127 sq meter
1 square foot = 144 sq in.

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